Lecture 7 - Collisions & the Pressure Tensor

09/19/17

*Corrections to HW #2

- $\nu_{ei} = \frac{4\pi n_i Z^2 e^4 \ln(\Lambda)}{m_e^2 v^3}$

  when he wrote the HW he used 2π, not 4π

- for #3 → do in steady state

  └ from the Coronal equilibrium model (Lec #3)

  $\dot{n}_e = n_e[n_H\langle\sigma_I v\rangle - n_p\langle\sigma_R v\rangle] = 0$ for steady state/equilibrium

from last time...

Deriving the Fluid Equations from the Vlasov Equation

The Vlasov equation with a collision operator can describe the dynamics of a plasma for Γ small (weakly coupled)

0^{th} Moment $(n=0)$:

$$\frac{\partial n_\alpha}{\vec{U}_\alpha} + \nabla\cdot(\underbrace{n_\alpha \vec{U}_\alpha}) = 0$$, the Continuity equation

$n_\alpha \vec{U}_\alpha \equiv \int d\vec{v}\,\vec{v} f_\alpha$ ⇒ defines $\vec{U}_\alpha$, the mean drift velocity of plasma species α

$n_\alpha \equiv \int d\vec{v} f_\alpha$

1^{st} Moment $(n=1)$:

$$m_\alpha n_\alpha\left[\frac{\partial}{\partial t}\vec{U}_\alpha + \vec{U}_\alpha\cdot\nabla\vec{U}_\alpha\right] = n_\alpha q_\alpha\left(\vec{E} + \frac{1}{c}\vec{U}_\alpha\times\vec{B}\right) - \nabla\cdot\overline{\overline{P}}_\alpha - \vec{R}_{\alpha\beta}$$

the Fluid Momentum Equation

pressure tensor

where

$\vec{R}_{\alpha\beta}$ = the drag between species α and β

$\overline{\overline{P}}_\alpha = m_\alpha \int d\vec{v}'\,\vec{v}'\vec{v}' f_\alpha$

└ the next moment is needed to get $\overline{\overline{P}}_\alpha$

This establishes the general pattern...

$(0^{th})\ \frac{\partial}{\partial t} n_\alpha + \nabla\cdot n_\alpha \vec{U}_\alpha = 0$

$(1^{st})\ \frac{\partial}{\partial t} m_\alpha n_\alpha \vec{U}_\alpha + (\ \ ) + \nabla\cdot\overline{\overline{P}}_\alpha = (\ \ )$

$(2^{nd})\ \frac{\partial}{\partial t}\overline{\overline{P}}_\alpha + (\ \ ) + \nabla\cdot[3^{rd}\text{ moment}] = (\ \ )$ etc.

⇒ No closure in a truly collisionless system

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Closure with Collisions

We know that collisions drive f_α toward a Maxwellian distribution. Thus, if collisions are strong enough we expect:

$$f_\alpha = \frac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}} e^{\overbrace{-(\vec{v}-\vec{U}_\alpha)^2 m_\alpha/2T_\alpha}^{\text{drifting Maxwellian}}} + \underbrace{\delta f_\alpha}_{\sim 1/\nu \to \text{small}}$$

Plugging into our definition for the pressure tensor

$$\overline{\overline{P}}_\alpha = m_\alpha \int d\vec{v}'\, \vec{v}'\vec{v}' f_\alpha$$

$\hookrightarrow \vec{v}' = \vec{v} - \vec{U}_\alpha$ — plasma velocity

random deviations from $\vec{U}_\alpha$

↓ in terms of tensor components ↓

$$P_{\alpha,ij} \cong m_\alpha \int d\vec{v}'\, \underbrace{v_i' v_j'}_{\text{because of this}}\, e^{-(v'^2 m_\alpha/2T_\alpha)} \frac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}}$$

This will be a diagonal tensor

└ all diagonal terms equal

$$P_{\alpha,ii} = m_\alpha \int \underbrace{d\vec{v}'}_{} v_i'^2 e^{-(v'^2 m_\alpha/2T_\alpha)} \frac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}}$$

* other two directions integrate out

$$= m_\alpha \int dv_i'\, v_i'^2 e^{-(v_i'^2 m_\alpha/2T_\alpha)} \frac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}}$$

define $s^2 \equiv \dfrac{m_\alpha v_i'^2}{2T_\alpha}$

↓ rewriting in terms of s^2 ↓

$$P_{\alpha,ii} = m_\alpha n_\alpha \left(\frac{2T_\alpha}{m_\alpha}\right)^{3/2} \frac{1}{(2\pi T_\alpha/m_\alpha)^{1/2}} \int_{-\infty}^{\infty} ds\, s^2 e^{-s^2}$$

$$\int_{-\infty}^{\infty} ds\, s^2 e^{-s^2} = -\frac{d}{d\epsilon} \int ds\, e^{-\epsilon s^2}$$

define $p^2 \equiv \epsilon s^2 \rightarrow ds = (\epsilon)^{-1/2} dp$

$$= -\frac{d}{d\epsilon}\frac{1}{\sqrt{\epsilon}}\underbrace{\int dp\, e^{-p^2}}_{\sqrt{\pi}}$$

$$\int_{-\infty}^{\infty} ds\, s^2 e^{-s^2} = \sqrt{\pi}/2$$

$$= \cancel{m_\alpha} n_\alpha \left(\frac{2T_\alpha}{\cancel{m_\alpha}}\right)^{3/2} \frac{1}{(2\pi T_\alpha/\cancel{m_\alpha})^{1/2}} \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$$

$$\Rightarrow P_{\alpha,ii} = n_\alpha T_\alpha \equiv \underbrace{P_\alpha}$$

where $\nabla\cdot\overline{\overline{P}}_\alpha = \nabla P_\alpha + \nabla\cdot\overline{\overline{\delta P}}_\alpha$

returning to the fluid momentum eq...

$$n_\alpha m_\alpha\left(\frac{\partial}{\partial t}\vec{U}_\alpha + \vec{U}_\alpha\cdot\nabla\vec{U}_\alpha\right) = n_\alpha q_\alpha\left(\vec{E} + \frac{1}{c}\vec{U}_\alpha\times\vec{B}\right) - \nabla P_\alpha - \underbrace{\nabla\cdot\overline{\overline{\delta P}}_\alpha}_{\text{viscous stress} \sim 1/\nu} - \vec{R}_{\alpha\beta}$$

BUT! We still need an equation for P_α

Second Moment ($n=2$): $\Rightarrow$ The Energy Equation

Take the Vlasov equation with collisions

$$\frac{\partial}{\partial t}\int d\vec{v}\, f_\alpha + \nabla\cdot\left(\int d\vec{v}\,\vec{v} f_\alpha\right) + \frac{q_\alpha}{m_\alpha}\int d\vec{v}\,\frac{\partial}{\partial\vec{v}}\cdot\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right) f_\alpha = \sum_\beta \int d\vec{v}\, C_{\alpha\beta}(f_\alpha)$$

$\downarrow$ multiply through by $\frac{1}{2}m_\alpha v^2$ $\downarrow$

$$\frac{\partial}{\partial t}\int d\vec{v}\,\frac{m_\alpha v^2}{2} f_\alpha + \nabla\cdot\left(\int d\vec{v}\,\frac{m_\alpha v^2}{2}\vec{v} f_\alpha\right) + \frac{q_\alpha}{\cancel{m_\alpha}}\int d\vec{v}\,\frac{\cancel{m_\alpha} v^2}{2}\frac{\partial}{\partial\vec{v}}\cdot\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right) f_\alpha$$

$$= \sum_\beta \int d\vec{v}\,\frac{m_\alpha v^2}{2} C_{\alpha\beta}(f_\alpha)$$

Recall f_α is a drifting Maxwellian $\quad f_\alpha = \dfrac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}} e^{-(\vec{v}-\vec{U}_\alpha)^2 m_\alpha/2T_\alpha}$

$$\vec{v}' = \vec{v} - \vec{U}_\alpha \longrightarrow \vec{v} = \vec{v}' + \vec{U}_\alpha$$

Plugging in term-by-term:

- $$\int d\vec{v}\,\frac{m_\alpha v^2}{2} f_\alpha = \frac{m_\alpha U_\alpha^2}{2}\underbrace{\int d\vec{v}\, f_\alpha}_{n_\alpha} + \frac{\cancel{m_\alpha}}{2}\underbrace{\int d\vec{v}'\,\vec{v}'^2 f_\alpha}_{3n_\alpha T_\alpha/\cancel{m_\alpha}}$$

  cross-terms go away: odd in $v' = 0$

$$= \frac{1}{2}n_\alpha m_\alpha U_\alpha^2 + \frac{3}{2}n_\alpha T_\alpha$$

- $$\int d\vec{v}\,\frac{m_\alpha v^2}{2}\vec{v} f_\alpha = \int d\vec{v}\,\frac{m_\alpha(\vec{v}' + \vec{U}_\alpha)^2}{2}(\vec{v}' + \vec{U}_\alpha) f_\alpha$$

$$= \frac{m_\alpha U_\alpha^2}{2}\vec{U}_\alpha \underbrace{\int d\vec{v}\, f_\alpha}_{n_\alpha} + \underbrace{\int d\vec{v}'\, \frac{m_\alpha v'^2}{2}\vec{v}' f_\alpha}_{\equiv \vec{Q}_\alpha} + \frac{\cancel{m_\alpha}}{2}\vec{U}_\alpha \underbrace{\int d\vec{v}'\, v'^2 f_\alpha}_{3 n_\alpha T_\alpha/\cancel{m_\alpha}}$$

$$+ \int d\vec{v}'\, \frac{\cancel{m_\alpha}}{\cancel{2}}\, \cancel{2}\, \underbrace{\vec{v}'\cdot\vec{U}_\alpha \vec{v}'}\, f_\alpha$$

only survives if two components of v' are the same

↳ lose two degrees of freedom

$$\int d\vec{v}'\, v'^2 f_\alpha - 2\text{D.O.F.} = n_\alpha T_\alpha/\cancel{m_\alpha}$$

$$= \frac{1}{2} n_\alpha m_\alpha U_\alpha^2 \vec{U}_\alpha + \vec{Q}_\alpha + \frac{3}{2} n_\alpha T_\alpha \vec{U}_\alpha + n_\alpha T_\alpha \vec{U}_\alpha$$

$$= \frac{1}{2} n_\alpha m_\alpha U_\alpha^2 \vec{U}_\alpha + \underbrace{\frac{5}{2} n_\alpha T_\alpha \vec{U}_\alpha}_{\text{enthalpy flux}} + \overbrace{\vec{Q}_\alpha}^{\text{heat flux}}$$

- $$q_\alpha \int d\vec{v}\, \underbrace{\frac{v^2}{2}\frac{\partial}{\partial \vec{v}}}_{\vec{v}} \cdot \left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right) f_\alpha = -q_\alpha \int d\vec{v}\, \vec{v}\cdot\left(\vec{E} + \cancelto{0}{\frac{1}{c}\vec{v}\times\vec{B}}\right) f_\alpha$$

motion of fluid in direction of the field

$$= -q_\alpha \vec{E} \underbrace{\int d\vec{v}\, \vec{v} f_\alpha}_{\vec{U}_\alpha n_\alpha} = -q_\alpha \vec{E}\cdot\vec{U}_\alpha n_\alpha$$

Putting it all together:

$$\frac{\partial}{\partial t}\left(\frac{1}{2} n_\alpha m_\alpha U_\alpha^2 + \frac{3}{2}\underbrace{n_\alpha T_\alpha}_{P_\alpha}\right) + \nabla\cdot\left(\frac{1}{2} n_\alpha m_\alpha U_\alpha^2 \vec{U}_\alpha + \frac{5}{2}\underbrace{n_\alpha T_\alpha}_{P_\alpha}\vec{U}_\alpha + \vec{Q}_\alpha\right)$$

$$- q_\alpha n_\alpha \vec{U}_\alpha\cdot\vec{E} = \underbrace{\sum_\beta \int d\vec{v}\, \frac{m_\alpha v^2}{2} C_{\alpha\beta}(f_\alpha)}$$

$$= -\left(\frac{\partial W}{\partial t}\right)_{\alpha\beta} = \text{rate of energy transfer from } \alpha \to \beta$$

*Note that

$$\left(\frac{\partial W}{\partial t}\right)_{\alpha\beta} = -\left(\frac{\partial W}{\partial t}\right)_{\beta\alpha}$$

and

$$\left(\frac{\partial W}{\partial t}\right)_{\alpha\alpha} = 0$$

} Collisions conserve total energy

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$$\frac{\partial}{\partial t}\left(\frac{1}{2}n_\alpha m_\alpha U_\alpha^2 + \frac{3}{2}P_\alpha\right) + \nabla\cdot\left(\frac{1}{2}n_\alpha m_\alpha U_\alpha^2 \vec{U}_\alpha + \frac{5}{2}P_\alpha\vec{U}_\alpha + \vec{Q}_\alpha\right)$$

$$- q_\alpha n_\alpha \vec{U}_\alpha\cdot\vec{E} = -\left(\frac{\partial W}{\partial t}\right)_{\alpha\beta}$$

→ We can simplify our expression by using the continuity and fluid momentum equations

- continuity:

$$\frac{\partial}{\partial t}n_\alpha + \nabla\cdot(n_\alpha\vec{U}_\alpha) = 0$$

- fluid momentum:

$$m_\alpha n_\alpha\left[\frac{\partial}{\partial t}\vec{U}_\alpha + \vec{U}_\alpha\cdot\nabla\vec{U}_\alpha\right] = n_\alpha q_\alpha\left(\vec{E} + \cancelto{0}{\frac{1}{c}\vec{U}_\alpha\times\vec{B}}\right) - \underbrace{\nabla\cdot\overline{\overline{P}}_\alpha}_{=\nabla P_\alpha + \nabla\cdot\delta P_\alpha} - \vec{R}_{\alpha\beta}$$

viscous stress term small as to be discarded; n/ν_{ei}

$$\Rightarrow \frac{3}{2}\left(\frac{\partial}{\partial t} + \vec{U}_\alpha\cdot\nabla\right)P_\alpha + \frac{5}{2}P_\alpha\nabla\cdot\vec{U}_\alpha = \underbrace{-\nabla\cdot\vec{Q}_\alpha}_{\text{often small}} + \vec{R}_{\alpha\beta}\cdot\vec{U}_\alpha - \left(\frac{\partial W}{\partial t}\right)_{\alpha\beta}$$

often thrown away

$-\vec{R}_{\alpha\beta}\cdot\vec{U}_\alpha$ = frictional heating of α due to drag with β

In a 3D system:

$\frac{3}{2}P_\alpha$ = the internal energy

→ $3 = n_f$, the # of degrees of freedom

→ $5 = n_f + 2$

So generalizing... (for any # of dimensions)

$$\Rightarrow \frac{n_f}{2}\left(\frac{\partial}{\partial t} + \vec{U}_\alpha\cdot\nabla\right)P_\alpha + \frac{(n_f+2)}{2}P_\alpha\nabla\cdot\vec{U}_\alpha = -\nabla\cdot\vec{Q}_\alpha + \vec{R}_{\alpha\beta}\cdot\vec{U}_\alpha - \left(\frac{\partial W}{\partial t}\right)_{\alpha\beta}$$

* In the case of no heat flux, collisional heating, or energy loss, the LHS = 0

⇒ This is the <u>Adiabatic Limit</u>

$$\frac{n_f}{\cancel{2}}\left(\frac{\partial}{\partial t} + \vec{U}_\alpha\cdot\nabla\right)P_\alpha + \frac{(n_f+2)}{\cancel{2}}P_\alpha\nabla\cdot\vec{U}_\alpha = 0$$

→

$$n_f\left(\frac{\partial}{\partial t}+\vec{U}_\alpha\cdot\nabla\right)P_\alpha+(n_f+2)P_\alpha\nabla\cdot\vec{U}_\alpha=0$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\vec{U}_\alpha\cdot\nabla \longrightarrow \frac{\partial}{\partial t}=\frac{d}{dt}-\vec{U}_\alpha\cdot\nabla$$

$$n_f\left[\left(\frac{d}{dt}-\cancel{\vec{U}_\alpha\cdot\nabla}\right)+\cancel{\vec{U}_\alpha\cdot\nabla}\right]P_\alpha+(n_f+2)P_\alpha\nabla\cdot\vec{U}_\alpha=0$$

$$n_f\frac{d}{dt}P_\alpha+(n_f+2)P_\alpha\nabla\cdot\vec{U}_\alpha=0$$

↓ divide through by n_f & P_α ↓

$$\frac{1}{P_\alpha}\frac{\cancel{n_f}}{\cancel{n_f}}\frac{d}{dt}P_\alpha+\underbrace{\frac{(n_f+2)}{n_f}}\frac{\cancel{P_\alpha}}{\cancel{P_\alpha}}\nabla\cdot\vec{U}_\alpha=0$$

$$* \quad \frac{(n_f+2)}{n_f}\equiv\gamma_s=\text{ratio of specific heats} \quad *$$

$$\Rightarrow \frac{1}{P_\alpha}\frac{d}{dt}P_\alpha+\gamma_s\nabla\cdot\vec{U}_\alpha=0 \qquad \text{eq. 1}$$

↳ very similar to the continuity equation,

$$\underbrace{\frac{\partial n_\alpha}{\partial t}}_{\frac{\partial}{\partial t}=\frac{d}{dt}-\vec{U}_\alpha\cdot\nabla}+\vec{U}_\alpha\cdot\nabla n_\alpha+n_\alpha\nabla\cdot\vec{U}_\alpha=0$$

$$\left(\frac{d}{dt}-\cancel{\vec{U}_\alpha\cdot\nabla}\right)n_\alpha+\cancel{\vec{U}_\alpha\cdot\nabla}n_\alpha+n_\alpha\nabla\cdot\vec{U}_\alpha=0$$

$$\frac{d}{dt}n_\alpha+n_\alpha\nabla\cdot\vec{U}_\alpha=0$$

↓ divide through by n_α ↓

$$\frac{1}{n_\alpha}\frac{d}{dt}n_\alpha+\frac{\cancel{n_\alpha}}{\cancel{n_\alpha}}\nabla\cdot\vec{U}_\alpha=0$$

↓ multiply by γ_s ↓

$$\gamma_s\left(\frac{1}{n_\alpha}\frac{d}{dt}n_\alpha+\nabla\cdot\vec{U}_\alpha\right)=0 \qquad \text{eq. 2}$$

Subtract eq. 2 from eq. 1

$$\frac{1}{P_\alpha}\frac{d}{dt}P_\alpha+\cancel{\gamma_s\nabla\cdot\vec{U}_\alpha}-\underbrace{\gamma_s\frac{1}{n_\alpha}\frac{d}{dt}n_\alpha}_{\text{move to other side}}-\cancel{\gamma_s\nabla\cdot\vec{U}_\alpha}=0$$

$$\frac{1}{P_\alpha}\frac{d}{dt}P_\alpha=\gamma_s\frac{1}{n_\alpha}\frac{d}{dt}n_\alpha$$

→

- rewrite in terms of the natural log

$\ln\left[\frac{1}{x}\frac{d}{dt}x\right] = \frac{d}{dt}\ln(x)$, $C\ln[x] = \ln(x^c)$

$\frac{d}{dt}\left[\ln(P_\alpha) - \ln(n_\alpha^{\gamma_s})\right] = 0$

$\ln(x) - \ln(y) = \ln(x/y)$

$\frac{d}{dt}\left(\frac{P_\alpha}{n_\alpha^{\gamma_s}}\right) = 0 \Rightarrow \frac{P_\alpha}{n_\alpha^{\gamma_s}} =$ constant in the frame of the moving plasma

NOW: Start talking about waves!

↳ go back to talk about waves in warm plasma, solve kinetic energy, look at $T \to$ small values, and some other stuff

→ Basically, how you do kinetic theory

*Start off in plasma systems with no magnetic fields ($\vec{B}=0$)

Electrostatic Waves in a Warm Plasma

→ No ambient magnetic field

From Lecture #2: Cold plasma limit dielectric function

plasma frequency, e^-

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \; ; \; \omega_{pe}^2 = \frac{4\pi n e^2}{m_e}, \; \omega_{pi}^2 = \frac{4\pi n e^2}{m_i}$$

for $\varepsilon \to 0$, $\omega = \pm\omega_{pe}$; $\omega_{pe} \gg \omega_{pi}$

Kinetic Theory for this? We want to explore what happens in a warm plasma — Do this by solving the Vlasov equation

We take the electric field to be small

↳ assuming small amplitude wave, we can throw away collisions

1st order in E

$f = f_0(x) + f_1 + \ldots$ Expand distribution f'n. as a series in the small parameter, electric field

0th order in E

Trying to solve...

$$\frac{\partial}{\partial t}f + \vec{v}\cdot\nabla f + \frac{q}{m}\vec{E}\cdot\frac{\partial}{\partial \vec{v}}f = 0$$

→ Solve order-by-order, taking wave amplitude to be small

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Zero Order

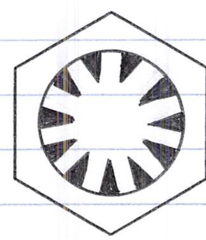
$$\frac{\partial}{\partial t} f_0 + \vec{v} \cdot \nabla f_0 = 0 \quad , \quad E_0 = 0$$

- $\frac{\partial}{\partial t} f_0 = 0 \rightarrow f_0$ is constant
- this is a homogeneous system: $\vec{v} \cdot \nabla f_0 = 0$

Take f_0 to be a Maxwellian distribution and require $-en_e + Zen_i = 0$

$\Rightarrow$ charge neutrality in initial state

First Order



- No magnetic field, $\vec{B}_1 = 0 \rightarrow \nabla \times \vec{E}_1 = 0$
- $\vec{E}_1 = -\nabla \varphi_1$

$$\varphi_1 = \mathrm{Re}\{\underbrace{\hat{\varphi}}_{\text{complex amplitude}} e^{i\overset{\text{wave vector}}{\vec{k}} \cdot \vec{x} - i\underbrace{\omega}_{\text{frequency; might be complex}} t}\}$$

Now linearize the Vlasov equation!

$$\frac{\partial f_1}{\partial t} + \vec{v} \cdot \nabla f_1 - \frac{q}{m} \nabla \varphi_1 \cdot \frac{\partial}{\partial \vec{v}} f_0 = 0$$

$\rightarrow$ all terms the same order in the expansion parameter; keeping only 1st order in amplitude

$$f_1 = \mathrm{Re}\{\hat{f} e^{i\vec{k} \cdot \vec{x} - i\omega t}\}$$

$\downarrow$ plugging in $\downarrow$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) \mathrm{Re}\{\hat{f} e^{i\vec{k} \cdot \vec{x} - i\omega t}\} - \frac{q}{m} \nabla \left[\mathrm{Re}\{\hat{\varphi} e^{i\vec{k} \cdot \vec{x} - i\omega t}\}\right] \cdot \frac{\partial}{\partial \vec{v}} f_0(\vec{v}) = 0$$

$\frac{\partial}{\partial t} \rightarrow -i\omega$ $\qquad$ $\nabla \rightarrow i\vec{k}$

$$\mathrm{Re}\left\{\underbrace{(-i\omega + i\vec{k} \cdot \vec{v})\hat{f} - \frac{q}{m} i \hat{\varphi} \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0}_{\text{This must be zero} \rightarrow \text{yields solution for } \hat{f}}\right\} = 0$$

(it's imaginary)

$$(-i\omega + i\vec{k} \cdot \vec{v})\hat{f} - \frac{q}{m} i \hat{\varphi} \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0 = 0$$

$$\hat{f} = \frac{q/m\, \hat{\varphi}\, \cancel{i} \vec{k} \cdot \partial/\partial \vec{v}\, f_0}{-\cancel{i}\omega + \cancel{i}\vec{k} \cdot \vec{v}} \Rightarrow \hat{f} = -\frac{q}{m} \hat{\varphi} \frac{1}{\underbrace{\omega - \vec{k} \cdot \vec{v}}} \vec{k} \cdot \frac{\partial}{\partial \vec{v}} f_0$$

$\omega - \vec{k} \cdot \vec{v}$ = frequency seen by moving particles