*Britt's Notes *Drake's notes

Lecture 21 – Plasma Waves & Instabilities   11/7/17

From last time...

MHD Waves, MHD

Linearized eqns: for sys that is initially homogen.

~~① $-\omega\rho_0\hat{u} = -\vec{k}\left(\hat{p} + \frac{\vec{B}_0\cdot\hat{B}}{4\pi}\right) + \frac{1}{4\pi}\vec{B}_0\cdot\vec{k}\hat{B}$ E field~~

② $\omega\hat{B} = -\vec{k}\cdot\vec{B}_0\hat{u} + \vec{B}_0\vec{k}\cdot\hat{u}$, $\vec{k}\cdot\hat{B}=0$ B-field & constr.

③ $\omega(\hat{p} - c_s^2\hat{\rho}) = 0$ pressure

④ $\omega\hat{\rho} = \rho_0\vec{k}\cdot\hat{u}$ mass density

⇒ 8 eqns and one constraint (all coupled)

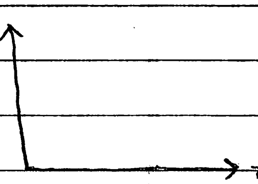
↳ 7 distinct values of ω for a given $\vec{k}$

Case I: $k_\parallel = 0$, $k_\perp \neq 0$

Magnetosonic wave

$$\omega^2 = k_\perp^2(c_s^2 + c_A^2)$$

sound speed   Alfvén speed



compressional mode, prop perp to B_0

$c_s^2 = \frac{\Gamma P_0}{\rho_0}$, $c_A^2 = \frac{B^2}{4\pi\rho_0}$ Γ = ratio of specific heats

⇒ 5 $\omega = 0$ modes

☆ Aside: next HW (what #), problem #1

- set of eqns in which ion dynamics are thrown out

↳ eqns for an electron fluid

Electron MHD Eqns

* impt for magnetosphere dynamics, etc.

$n_{0i} = n_{0e} = $ const

electron eqn of motion: (ignore pressure; it's also const)

~~$m_e n_0 \frac{d}{dt}\vec{v}_e = -n_0 e\vec{E} - \frac{e}{c}n_0\vec{v}_e\times\vec{B}$~~

- can write in terms of J since ions ~ stationary

$\vec{J} = -n_e e v_e$

· plug in J,
multiply top & bottom by e ?

$$\underbrace{-\frac{m_e}{e}\frac{d}{dt}\vec{J}}_{\text{current}} = -n_0 e\vec{E} - \frac{e}{c}n_0\vec{v}_e\times\vec{B}$$

└ electric field

solving...

$$\frac{1}{c}\frac{\partial}{\partial t}\vec{B} + \nabla\times\vec{E} = 0\ ,\quad \vec{J} = \nabla\times\vec{B}\left(\frac{4\pi}{c}\right)$$

} complete indp. set of eqns

⇒ yields "whistler waves"

· divide by n_0 to get char. scale

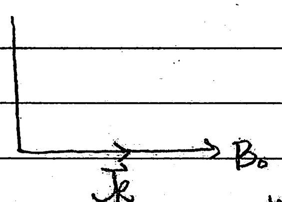
↳ $\frac{m_e}{n_0 e^2} \Rightarrow \frac{c^2}{\omega_{pe}^2}$ } scale length

where $c/\omega_{pe} = d_e$, the e^- skin depth

Case II: $k_\parallel \neq 0$, $k_\perp = 0$

need various comp. of $\vec{u}$ & $\vec{B}$

└ project comp. in specific directions



· Take $\vec{k}\cdot$ ① ; where we know $\vec{k}\cdot\hat{\vec{B}} = 0$

(10) $$\omega\rho_0\vec{k}\cdot\hat{\vec{u}} = k_\parallel^2\left(\hat{p} + \cancel{\frac{\vec{B}_0\cdot\hat{\vec{B}}}{4\pi}}\right)$$ → 0 must also be 0 b/c k in $\vec{B}_0$ direction ($\vec{k}\cdot\hat{\vec{B}}=0$)

· Take $\vec{k}\times$ ① to elim 1st RHS term

(11) $$\omega\rho_0\vec{k}\times\hat{\vec{u}} = -\frac{1}{4\pi}k_\parallel B_0\underbrace{\vec{k}\times\hat{\vec{B}}}$$

take $\vec{k}\times$ ② to determine this

(12) $$\omega\vec{k}\times\hat{\vec{B}} = \underbrace{-\vec{k}\cdot\vec{B}_0}_{-k_\parallel B_0}\ \vec{k}\times\hat{\vec{u}}$$

These form a closed sys of eqns w/ ③ & ④:

(13) $\omega(\hat{p} - c_s^2\hat{\rho}) = 0$

└ if $\hat{p} - c_s^2\hat{\rho} \neq 0$, $\omega = 0$

(14) $\omega\hat{\rho} = \rho_0\vec{k}\cdot\hat{\vec{u}}$

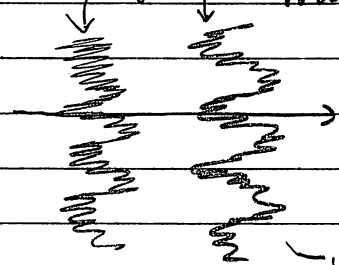
What modes does this give us?

ⓐ: $\hat{P} = 0$, $\hat{\rho} \neq 0$,

($\Rightarrow$ entropy mode, from eq. ⑬

$\rightarrow$ means $n_0 \hat{T} + \hat{n} T_0 = 0$, where $T \neq 0$

if n goes up, T goes down to maintain const press (const $P \rightarrow$ no sound wave)



*becomes purely damped mode

plasmas really quickly smooth out temp. $\rightarrow$ not a realistic mode.

$\Rightarrow$ doesn't really exist in nature (related to temp. variation along B_0 ẑ, unrealistic thermal conduction)

ⓑ: ⑩, ⑬, ⑭ form closed set that yield sound waves

$$\omega \cancel{\rho_0} \vec{k} \cdot \hat{\vec{u}} = k_\parallel^2 C_s^2 \frac{1}{\omega} \cancel{\rho_0} \vec{k} \cdot \hat{\vec{u}}$$

$$\rightarrow \omega^2 = k_\parallel^2 C_s^2$$

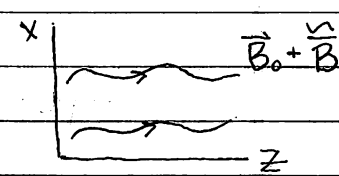
*comp. moving along $\vec{B}$-field $\quad \underbrace{\omega = \pm k_\parallel C_s}$ 2 modes assoc w/ this!

ⓒ: Alfvèn Waves — ⑪ & ⑫ $\quad$ Note: Alfven waves do not carry entropy

$$\omega \rho_0 \cancel{\vec{k} \times \hat{\vec{u}}} = + \frac{1}{4\pi} \frac{k_\parallel B_0}{\omega} (+) B_0 k_\parallel \cancel{\vec{k} \times \hat{\vec{u}}}$$

$$\omega^2 = k_\parallel^2 \frac{B_0^2}{4\pi\rho_0} = k_\parallel^2 C_A^2$$

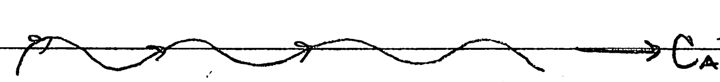
└ 2 modes here as well!



Two Polarizations

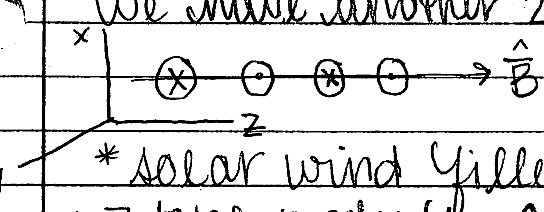
$\hat{B}_x, \hat{u}_x \neq 0$,

$\hat{B}_y, \hat{u}_y \neq 0$



perturbations travel @ Alfvèn speed

We have another 2 $\rightarrow$ in/out!



└ i.e. $\pm k_\parallel C_A$ for each polarization

*solar wind filled w/ Alfvèn waves!

$\Rightarrow$ 7 total modes for Case II

Case III – The General Case $k_{\parallel} \neq 0,\ \vec{k}_{\perp} \neq 0$

Goal: How to solve these w/out doing a ton of algebra

INSERT! Pg. 9

③, ④ unchanged, but must deal w/ coupled vector eqns ① & ②

↳ we know $\vec{k}\cdot\hat{\vec{u}}$ an impt value but do not want to break down $\vec{k}$ or $\vec{u}$ into x, y, z comp.

Strategy on how to solve eqns:

- Take $\vec{k}\cdot$①
  gives one comp of $\hat{\vec{u}}$ (elim 3rd term RHS)
- Take $\vec{B}_0\cdot$①
- Take $(\vec{k}\times\vec{B}_0)\cdot$① (elim 1st term RHS)

gives all 3 comp of $\vec{u}$

- Still have $\vec{k}\cdot\hat{\vec{B}}=0$
- $\vec{B}_0\cdot$②
- $(\vec{k}\times\vec{B}_0)\cdot$②, $B_0\vec{k}\cdot\hat{\vec{u}}$ term goes away

All covered in the insert

☆ Algebra for this in online notes

Result:

4 modes

$$\omega^2 = \frac{k^2 C_{ms}^2 \pm \sqrt{k^4 C_{MS}^4 - 4k^2 C_S^2 k_{\parallel}^2 C_A^2}}{2}$$

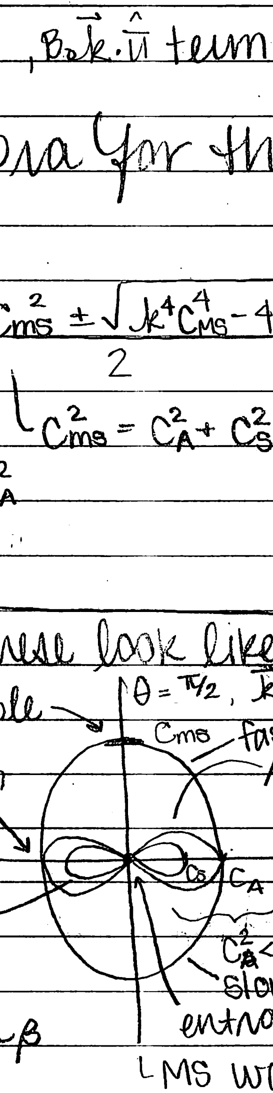
magnetosonic wave

$C_{ms}^2 = C_A^2 + C_S^2$

$$\omega^2 = k_{\parallel}^2 C_A^2$$

Alfvén Wave (comes out on its own even in general angle case)

What do these look like? – plot phase velocity of wave as function of θ



$\beta = \frac{8\pi P_0}{B_0^2}$

$\beta \ll 1$

└ plasma β

entropy mode just a point (no real interest in this mode)

└ MS wave in perp direc (no $k_{\parallel}$; have $\omega=0$ & MS modes)

What happens to C_{ms} as $\theta \to 0$ (when $\vec{k}$ along $\vec{B_0}$)?

Parallel propagation

$\cdot\ \vec{k}_\perp = 0$

$$\omega^2 = \frac{1}{2}\left[k_\parallel^2 C_{ms}^2 \pm \sqrt{\underbrace{k_\parallel^4 \underbrace{C_{ms}^4}_{(C_A^2+C_S^2)^2} - 4k_\parallel^4 C_S^2 C_A^2}_{k_\parallel^4 (C_A^2 - C_S^2)^2}}\,\right]$$

$-$ sign gives 0-freq wave (sound wave)

$$= \frac{k_\parallel^2 (C_A^2 + C_S^2) \pm k_\parallel^2 (C_A^2 - C_S^2)}{2}$$

$\oplus = \frac{1}{2} k_\parallel^2 [2C_A^2]$

$\ominus = \frac{1}{2} k_\parallel^2 [2C_S^2]$

$\Rightarrow k_\parallel^2 C_A^2$ MS wave becomes Alfven wave

$C_A^2 < C_S^2$ corresponds to

$$\frac{B_0^2}{4\pi \rho_0} > \frac{\Gamma P_0}{\rho_0} \Rightarrow \beta < 1,\ \beta = \frac{8\pi P_0}{B_0^2}$$

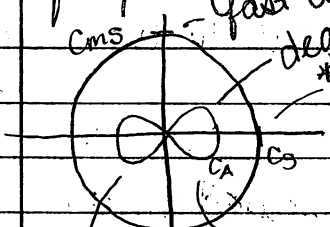
$\theta \to \pi/2$?

$\perp$ Propagation

$\cdot\ k_\parallel = 0$

$\omega^2 = k^2 C_{ms}^2$ and 0

for $\beta >> 1$:



- fast wave
- degenerate modes
- *sound waves really susceptible to damping (compressional modes & ms modes readily damped)
  - have resonant interactions in collisionless case
- Alfven wave fully inside sound wave in this case
- *Alfven waves much harder to damp

BUT: We had no energy source — no growth or damping

NOW: Add free energy — generates instabilities in these waves

└ instabilities lead to growth

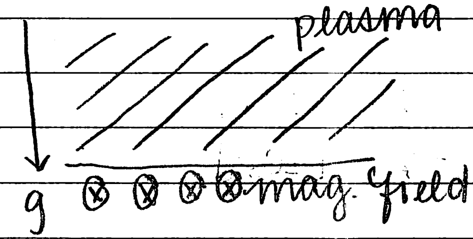
Instabilities dep. on type of free energy!

$\longrightarrow$

Ideal MHD Stability

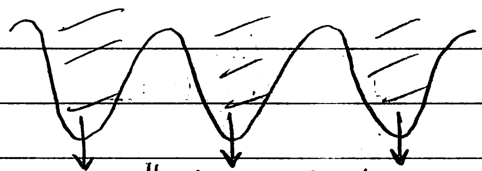
Driver — type of free energy

(1) Gravitational: Rayleigh Taylor
↳ heavy material supported by light material



unstable to Rayleigh Tay.

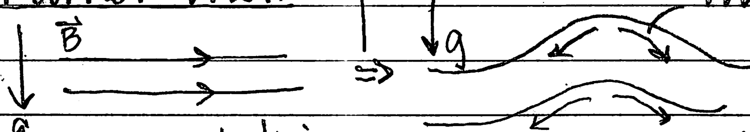
⇓ evolves to



└ "fingers" dragged down
B field does nothing to stop

Parker Inst

apply perturbation wavevectors to undulate field lines



mass density draining down "along undulated field lines into magnetic valleys"

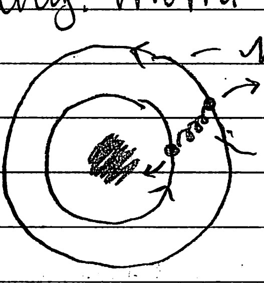
*this is a buoyancy inst.

Magnetorotational Instability (MRI)

*ang. mom. transport in accretion disks

e.g. accretion of matter in a disc around a BH or NS

— mat. forms flows

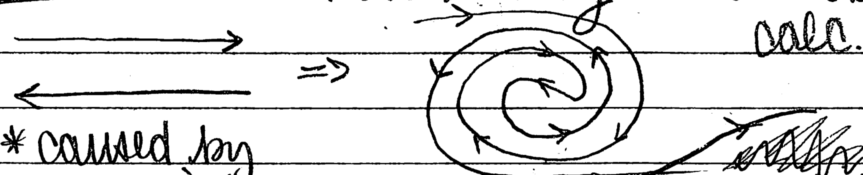


"spring coupling" moves ang mom. out and matter inward to accrete

*From A680 — The inner particle is moving faster than its partner. The inner particle is then slowed (think "pulled back") by the outer, giving some of its ang mom. to the outer, such that it must move inward and accrete. Outer also more stable now on closer orbit.

predicts the onset of instability and transition to turbulent flow in fluids of different densities moving @ different speeds

Driver ② <u>Flow:</u> Kelvin Helmholtz ** will be HW prob to calc. growth rate

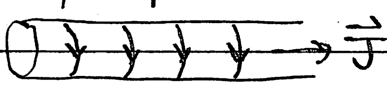


* caused by shear in flow (velocity difference across ~~the interface between~~ two fluids)

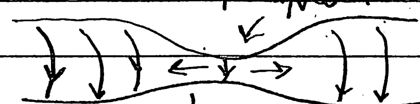
wraps in on self to form vortices of instability, e.g. solar wind, planetary magnetosphere

Driver ③ <u>Magnetic Field:</u>

Z-pinch

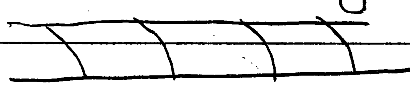


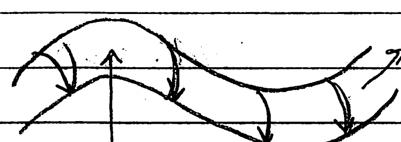
"sausage instability"
"necking down"



plasma flow driven by magnetic tension increase at pinch

kink instability

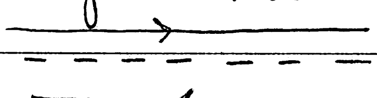




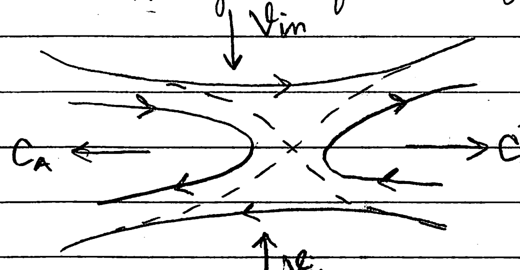
* Where is plasma flow??

magnetic forces larger inside the kink

magnetic reconnection



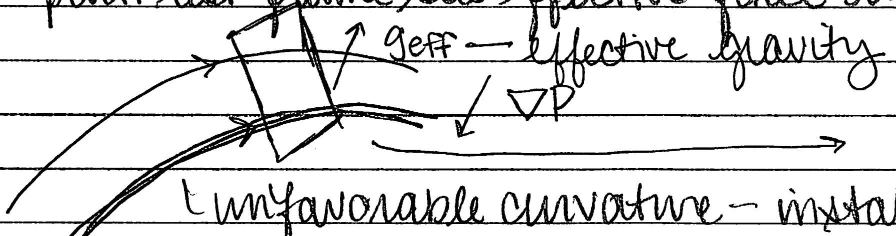
e.g. solar & planetary magnetospheres

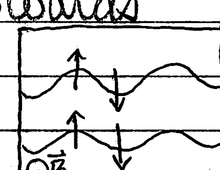


plasma shoots out @ Alfvén speed

D④ Pressure

· part. rest frame sees effective force outwards



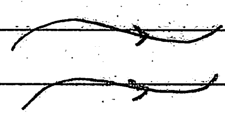


$\downarrow \nabla P$

unfavorable curvature — instability like R-Tay. b/c of orientation

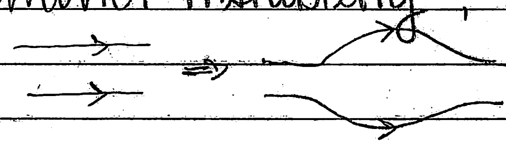
→ drift waves, * important in magnetospheres, fusion
caused by effective
curvature like this?

→ Pressure anisotropy
$P_{\parallel} \neq P_{\perp}$ drives instability
· firehose instability ("thrashes around")



$P_{\parallel} > P_{\perp}$, $\vec{B} \neq 0$
└ stuff going along faster than containment

· mirror instability, $P_{\perp} > P_{\parallel}$ forms spontaneous



$B \neq 0$ mag. mirrors
(recall from HW)

What if no ambient B?
Weibel instability ~~$B \neq 0$?~~ $B = 0$
$P_{xx} \neq P_{yy} \Rightarrow$ spontaneous generation of $\vec{B}$
* Important in shock waves

NEXT: - details of Rayleigh Taylor
- $P_{\parallel} > P_{\perp}$
- EM waves not descr. by MHD

INSERT: Case III – The General Case

From ③ & ④

③ $\omega(\hat{P} - C_s^2 \hat{\rho}) = 0$

④ $\omega \hat{\rho} = \rho_0 \vec{k} \cdot \hat{\vec{u}}$

$\Bigg\} \quad \hat{P} = C_s^2 \frac{1}{\omega} \rho_0 \vec{k} \cdot \hat{\vec{u}}$

↑ both unchanged in this case

SPACE ⇒ We must deal with the coupled vector eqns ① & ②

└ we know $\vec{k} \cdot \hat{\vec{u}}$ is an important value, but we do not want to break down $\vec{k}$ or $\hat{\vec{u}}$ into x, y, z components

– Take $\vec{k} \cdot$ ①

$$\vec{k} \cdot (-\omega \rho_0 \hat{\vec{u}}) = \vec{k} \cdot \Big[-\vec{k} \Big(\hat{P} + \frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi} \Big) + \underbrace{\frac{1}{4\pi} \vec{B}_0 \cdot \vec{k} \hat{\vec{B}}}_{\text{this term is eliminated; } \vec{k} \cdot \hat{\vec{B}} = 0} \Big]$$

$$+\omega \rho_0 \vec{k} \cdot \hat{\vec{u}} = +k^2 \Big(\hat{P} + \frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi} \Big)$$

└ plug in above value from ③ & ④

$$\omega \rho_0 \vec{k} \cdot \hat{\vec{u}} = k^2 \Big(C_s^2 \frac{1}{\omega} \rho_0 \vec{k} \cdot \hat{\vec{u}} + \frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi} \Big)$$

↓ multiply through by ω ↓

$$⑮ \quad \omega^2 \rho_0 \vec{k} \cdot \hat{\vec{u}} = k^2 \Big(C_s^2 \rho_0 \vec{k} \cdot \hat{\vec{u}} + \omega \frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi} \Big)$$

– Take $\vec{B}_0 \cdot$ ①

$$\vec{B}_0 \cdot (-\omega \rho_0 \hat{\vec{u}}) = \vec{B}_0 \cdot \Big[-\vec{k} \Big(\hat{P} + \frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi} \Big) + \frac{1}{4\pi} \vec{B}_0 \cdot \vec{k} \hat{\vec{B}} \Big]$$

$$\omega \rho_0 \hat{\vec{u}} \cdot \vec{B}_0 = \vec{k} \cdot \vec{B}_0 \Big(\hat{P} + \cancel{\frac{\vec{B}_0 \cdot \hat{\vec{B}}}{4\pi}} \Big) - \cancel{\frac{1}{4\pi} \vec{k} \cdot \vec{B}_0 (\vec{B}_0 \cdot \hat{\vec{B}})}$$

└ plug in for $\hat{P}$

$$\omega \cancel{\rho_0} \hat{\vec{u}} \cdot \vec{B}_0 = C_s^2 \frac{1}{\omega} \cancel{\rho_0} (\vec{k} \cdot \vec{B}_0)(\vec{k} \cdot \hat{\vec{u}})$$

$$⑯ \quad \omega \hat{\vec{u}} \cdot \vec{B}_0 = \frac{C_s^2}{\omega} (\vec{k} \cdot \vec{B}_0)(\vec{k} \cdot \hat{\vec{u}})$$

→

- Take $(\vec{k}\times\vec{B}_0)\cdot$ ①

$$(\vec{k}\times\vec{B}_0)\cdot(-\omega\rho\hat{\vec{u}}) = (\vec{k}\times\vec{B}_0)\cdot\left[-\vec{k}\left(\frac{C_s^2}{\omega}\rho_0\vec{k}\cdot\hat{\vec{u}} + \frac{\vec{B}_0\cdot\hat{\vec{B}}}{4\pi}\right) + \frac{1}{4\pi}\vec{B}_0\cdot\vec{k}\hat{\vec{B}}\right]$$

$$\omega\rho_0\hat{\vec{u}}\cdot(\vec{k}\times\vec{B}_0) = \frac{C_s^2}{\omega}\rho_0\cancelto{0}{\vec{k}(\vec{k}\cdot\hat{\vec{u}})\cdot(\vec{k}\times\vec{B}_0)} + \frac{\vec{k}}{4\pi}\cancelto{0}{(\vec{B}_0\cdot\hat{\vec{B}})\cdot(\vec{k}\times\vec{B}_0)} - \frac{1}{4\pi}(\vec{B}_0\cdot\vec{k}\hat{\vec{B}})\cdot(\vec{k}\times\vec{B}_0)$$

$$\vec{k}\cdot\hat{\vec{B}}=0$$

⑰ $$\omega\rho_0\hat{\vec{u}}\cdot(\vec{k}\times\vec{B}_0) = \frac{-1}{4\pi}(\vec{k}\cdot\vec{B}_0)(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{B}}$$

⑮, ⑯, and ⑰ give all three components of $\hat{\vec{u}}$. We now must determine the three components of $\vec{k}$

- Still have $\vec{k}\cdot\hat{\vec{B}}=0$
- Take $\vec{B}_0\cdot$ ②

$$\vec{B}_0\cdot(\ \omega\hat{\vec{B}}) = \vec{B}_0\cdot(-\vec{k}\cdot\vec{B}_0\hat{\vec{u}} + \vec{B}_0\vec{k}\cdot\hat{\vec{u}})$$

⑱ $$\omega\hat{\vec{B}}\cdot\vec{B}_0 = (-\vec{k}\cdot\vec{B}_0)\cdot(\vec{B}_0\cdot\hat{\vec{u}}) + B_0^2(\vec{k}\cdot\hat{\vec{u}})$$

*this does not match online notes, pg. 145 (Lec 11a).

- Take $(\vec{k}\times\vec{B}_0)\cdot$ ②

$$(\vec{k}\times\vec{B}_0)\cdot(\omega\hat{\vec{B}}) = (\vec{k}\times\vec{B}_0)\cdot(-\vec{k}\cdot\vec{B}_0\hat{\vec{u}} + \vec{B}_0\vec{k}\cdot\hat{\vec{u}})$$

$$\omega(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{B}} = (-\vec{k}\cdot\vec{B}_0)(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{u}} + \vec{B}_0\vec{k}\cancelto{0}{(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{u}}}$$

⑲ $$\omega(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{B}} = (-\vec{k}\cdot\vec{B}_0)(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{u}}$$

Our constraint along with ⑱ and ⑲ give all three components of $\vec{k}$. Having this full set of equations and constraints, we can solve for the modes of this general case.

→ Begin w/ Eq. ⑮

Eq. ⑱ $$\omega\vec{B}_0\cdot\hat{\vec{B}} = (-\vec{k}\cdot\vec{B}_0)\cdot(\vec{B}_0\cdot\hat{\vec{u}}) + B_0^2(\vec{k}\cdot\hat{\vec{u}})$$

$$\omega^2\rho_0(\vec{k}\cdot\hat{\vec{u}}) = k^2\left(C_s^2\rho_0\vec{k}\cdot\hat{\vec{u}} + \omega\overbrace{\frac{\vec{B}_0\cdot\hat{\vec{B}}}{4\pi}}\right)$$

↑ divide through by ρ_0

$$\vec{B}_0\cdot\hat{\vec{u}} = \frac{1}{\omega}\text{⑯} = \frac{1}{\omega}\left[\frac{C_s^2}{\omega}(\vec{k}\cdot\vec{B}_0)(\vec{k}\cdot\hat{\vec{u}})\right]$$

$$\omega^2(\vec{k}\cdot\hat{\vec{u}}) = k^2\left[C_s^2(\vec{k}\cdot\hat{\vec{u}}) + \frac{1}{4\pi\rho_0}\left(B_0^2(\vec{k}\cdot\hat{\vec{u}}) - \underset{(\vec{k}\cdot\vec{B}_0)}{\wedge}(\vec{B}_0\cdot\hat{\vec{u}})\right)\right]$$

$$\omega^2(\cancel{\vec{k}\cdot\hat{\vec{u}}}) = k^2\left[C_S^2(\cancel{\vec{k}\cdot\hat{\vec{u}}}) + \frac{1}{4\pi\rho_0}\left(B_0^2(\cancel{\vec{k}\cdot\hat{\vec{u}}}) - \frac{C_S^2}{\omega^2}(\vec{k}\cdot\vec{B}_0)^2(\cancel{\vec{k}\cdot\hat{\vec{u}}})\right)\right]$$

☆ C_A^2 * In what Lec did we define this? #20?

$$\omega^2 = k^2C_S^2 + \frac{k^2 B_0^2}{4\pi\rho_0} - \frac{C_S^2}{4\pi\rho_0\,\omega^2}(\vec{k}\cdot\vec{B}_0)^2 k^2$$

☆ $k_{\parallel}^2 C_A^2$; from Lec #20, case II

$$\omega^2 - \underbrace{k^2(C_S^2 + C_A^2)}_{} + \frac{k^2C_S^2 k_{\parallel}^2 C_A^2}{\omega^2} = 0$$ ← multiply through

C_{MS}^2 = magnetostatic wave speed ($= C_F$, fast wave)
$= C_S^2 + C_A^2$

$$(\omega^2 - k^2C_{ms}^2)\omega^2 + k^2C_S^2k_{\parallel}^2C_A^2 = 0$$

↳ quadratic equation

$$\Rightarrow \underbrace{\omega^2}_{4\text{ modes}} = \frac{k^2C_{ms}^2 \pm \sqrt{k^4C_{ms}^4 - 4k^2C_S^2k_{\parallel}^2C_A^2}}{2}$$ magnetosonic waves

BUT! We still have two unused equations
→ Begin w/ Eq. (17)

$$\omega\rho_0\hat{\vec{u}}\cdot(\vec{k}\times\vec{B}_0) = \frac{-1}{4\pi}(\vec{k}\cdot\vec{B}_0)\underbrace{(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{B}}}_{=\frac{1}{\omega}(19) = \frac{-1}{\omega}(\vec{k}\cdot\vec{B}_0)(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{u}}}$$

divide out

$$\omega\rho_0\cancel{\hat{\vec{u}}\cdot(\vec{k}\times\vec{B}_0)} = \frac{1}{4\pi\omega}(\vec{k}\cdot\vec{B}_0)^2\cancel{(\vec{k}\times\vec{B}_0)\cdot\hat{\vec{u}}}$$

multiply through

$$\omega^2 = \frac{(\vec{k}\cdot\vec{B}_0)^2}{4\pi\rho_0}$$ $k_{\parallel}^2C_A^2$

$\Rightarrow \omega^2 = k_{\parallel}^2 C_A^2$ Alfvén waves

*Note: this comes out on its own even in the general angle case
– This is a shear Alfvén, where $\hat{P}=0$ and $\hat{\vec{B}}\cdot\vec{B}_0 = 0$