Lecture 1 - Introduction

08/29/17

Administrative stuff:

Homeworks typically due Thursdays

→ homework assigned every week

→ must do every homework (no exams!)

→ open to delaying an assignment if material not sufficiently covered

*keep track of how much time each homework takes

Introduction → The plasma state - basic parameters

What is a plasma?

∴ A plasma is a collection of charged particles that interact dominantly through the Coulomb and magnetic forces

How are plasmas produced?

- A plasma is produced when a neutral gas is heated until inter-atomic collisions become sufficiently violent that they detach electrons from colliding atoms
- Plasmas resulting from the ionization of neutral gases consist of myriads of positive and negative charge carriers whose relative numbers are in the inverse proportion to the magnitude of their individual charges
  - ↳ In this case, the oppositely-charged fluids tend to neutralize one another on macroscopic length scales and are termed "quasi-neutral"

Examples of plasmas:

→ In space!

*In earlier epochs of the Universe, all (baryonic) matter was in the plasma state. In the present epoch, most (baryonic) matter remains in this state

- the Solar System is permeated with plasma in the form of

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solar wind
- the Earth is completely surrounded by plasma trapped within its magnetic field

— interstellar gas
— stellar & solar coronas
— magnetospheres of compact objects (e.g., BH, NS)
— ionosphere
— planetary magnetospheres

→ On Earth!
— flames (weakly ionized)
— fusion experiments
— materials processing reactions (creating chips)
— charged particle beams (accelerators)
— light sources
— laser-plasma

Characteristics of plasmas:
- typically less dense than condensed matter
- typically hotter than room temperature (atoms must be ionized)
- typically a gas (although not always); "gas-like"
- passes mobile charge carriers
  - ↳ Implication: acts like a conductor; a metal

Basic Parameters & Units

In the plasma formulary → use cgs-esu

Density: $\underbrace{n_{e,i}}_{\text{electrons or ions}}$, cm^{-3} # of particles per cubic cm/ number density

Charge: stat-Coulombs

$1\ \underset{[C]}{\text{Coulomb}} = 3\times10^{9}\ \underset{[sC]}{\text{statCoulombs}} = 3\times10^{9}$ esu

$e = 4.8032\times10^{-10}\ sC$

→

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Force: $|\vec{F}_{12}| \sim \frac{q_1 q_2}{r_{12}^2} \sim$ dynes

* $\vec{F} = \underset{\sim}{F}$ in Drake notation

→ Expressing the force in dynes can be a useful simplification to Maxwell's equations

Energy: ergs

$$1\,\text{erg} = 1\,\frac{g \cdot cm^2}{s^2} = 10^{-7}\,J$$

Maxwell's Equations [in esu]:

$$\nabla \cdot \vec{B} = 0 \qquad \frac{1}{c}\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{E} = 4\pi\rho \qquad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

ρ = charge/volume

*recall: $c = 3 \times 10^{10}$ cm/s in cgs

- $\vec{B}$ = magnetic field [gauss]
  1 tesla [T] = 10^4 gauss [G]
- $\vec{E}$ = electric field [stat volts/cm]
  1 statvolt [sV] = 299.792458 volts [V]
  just c [m/s] divided by 10^6
  1 statvolt = 1 erg/esu = 1 erg/sC

→ the nice thing about choosing cgs-esu is that $\vec{E}$ & $\vec{B} \sim$ the same units

It follows that

$$\vec{F} = q\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right)$$

Electric potential: Q = statvolts

1 statvolt [sV] $\sim \frac{1}{3} \times 10^{-2}$ volts [V]

Temperature: energy per degree of freedom

Boltzmann constant = 1.3806×10^{-16} erg/°Kelvin

$$U = \frac{1}{2} k_B T$$

ergs

°Kelvin (note: Drake does not like °K)

a.k.a. thermal equilibrium

- express temperature in units of energy

$k_B T \rightarrow T$ [ergs]

$\Rightarrow U = \frac{1}{2} T$

- We usually measure temperature in eV

$1\,eV = 1.6022 \times 10^{-14}$ J

$= 1.6022 \times 10^{-12}$ ergs

$= 1.16 \times 10^{4}$ °Kelvin

$\hookrightarrow 1\,eV \sim 10^{4}$ °Kelvin

* we will use both ergs & eV for energy

Kinetic Temperature:

Consider an idealized plasma consisting of an equal number of electrons, with mass m_e and charge $-e$, and ions, with mass m_i and charge $+e$

kinetic temperature (measure in energy units)

$T_s \equiv \frac{1}{3} m_s \underbrace{\langle v_s^2 \rangle}_{\text{ensemble average}}$ for the plasma species

not necessarily in therm. eq.

– We can estimate typical particle speeds in terms of the so-called thermal speed

$\hookrightarrow$ assuming that both ions and electrons are characterized by the same temperature, T, which is by no means always the case in plasmas

$$v_{ts} \equiv \left(\frac{2T}{m_s}\right)^{1/2}$$

– Incidentally, the ion thermal speed is usually far slower than the electron thermal speed

$$v_{ti} \sim \left(\frac{m_e}{m_i}\right)^{1/2} v_{te}$$

* n and T are generally functions of position in a plasma

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APPROXIMATE MAGNITUDES IN SOME TYPICAL PLASMAS

Plasma Type	n cm^{-3}	T eV	ω_{pe} sec^{-1}	λ_D cm	$n\lambda_D{}^3$	ν_{ei} sec^{-1}
Interstellar gas	1	1	6×10^4	7×10^2	4×10^8	7×10^{-5}
Gaseous nebula	10^3	1	2×10^6	20	8×10^6	6×10^{-2}
Solar Corona	10^9	10^2	2×10^9	2×10^{-1}	8×10^6	60
Diffuse hot plasma	10^{12}	10^2	6×10^{10}	7×10^{-3}	4×10^5	40
Solar atmosphere, gas discharge	10^{14}	1	6×10^{11}	7×10^{-5}	40	2×10^9
Warm plasma	10^{14}	10	6×10^{11}	2×10^{-4}	8×10^2	10^7
Hot plasma	10^{14}	10^2	6×10^{11}	7×10^{-4}	4×10^4	4×10^6
Thermonuclear plasma	10^{15}	10^4	2×10^{12}	2×10^{-3}	8×10^6	5×10^4
Theta pinch	10^{16}	10^2	6×10^{12}	7×10^{-5}	4×10^3	3×10^8
Dense hot plasma	10^{18}	10^2	6×10^{13}	7×10^{-6}	4×10^2	2×10^{10}
Laser Plasma	10^{20}	10^2	6×10^{14}	7×10^{-7}	40	2×10^{12}

The Plasma Parameter

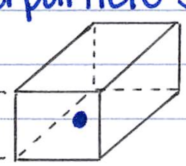
→ Important for type of plasma ←

Want to answer: How important are individual Coulomb forces/ interactions vs. many-particle interactions?

i.e. is the particle interacting more with adjacent neighbors or beyond with a broad group?

Consider a plasma of density n, temperature T:

→ Interparticle spacing?

r_s { 

One particle in a cube of side r_s

$n r_s^3 = 1$ so $r_s = 1/n^{1/3}$

→ Coulomb energy associated with adjacent particles — want to compare this with the energy associated with temperature

$U_e \sim e/r_s$

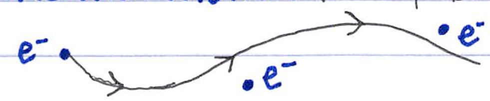
→ Typical thermal energy

$$U_T \sim \frac{3}{2} T$$

From these, we get the <u>Plasma Parameter</u>

$$\underbrace{U_e}_{\text{Coulomb energy}} \Big/ \underbrace{U_T}_{\text{Thermal energy}} \simeq \frac{e^2}{r_s T} \equiv \Gamma \quad \text{the plasma parameter}$$

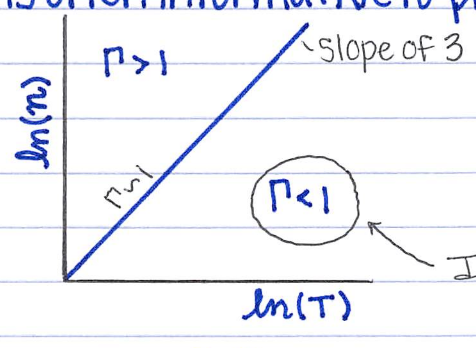
- if $\Gamma < 1$: nearby particles are weakly correlated – corresponds to thermal motion (many-body interactions dominate)



Charged particles move ballistically, suffering only slight deflections due to individual interactions

- if $\Gamma > 1$: nearby particles are strongly correlated – corresponds to motion from Coulomb forces (single particle interactions dominate)
- $\Gamma > 2$: gas/liquid phase transition
- $\Gamma > 180$: liquid/crystal phase transition

It is often informative to plot $\ln(T)$ vs. $\ln(n)$:



In this class, we will mostly work in this region

$$\Gamma = \frac{e^2 n^{1/3}}{T}$$

for $\Gamma = 1 = \frac{e^2 n^{1/3}}{T}$

$\hookrightarrow T \sim e^2 n^{1/3}$

$\ln(T) \sim \frac{1}{3} \ln(n)$

*See Fig. 11.2 at the end of this lecture for this plot in relation to different plasma systems

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Debye Length — corresponds to screening distance in a plasma

$q \bullet |\leftarrow \lambda_D \rightarrow|$ beyond this length, the particle is shielded

$$\lambda_D^2 \equiv \frac{T}{4\pi n e^2}$$

note that the Debye length is independent of mass, and therefore generally comparable for different species

Want to rewrite the plasma parameter in terms of λ_D

$$\lambda_D^2 = \frac{T}{4\pi n e^2} \rightarrow T = 4\pi n e^2 \lambda_D^2$$

↓ plugging in ↓

$$\Gamma = \frac{e^2 n^{1/3}}{T} = \frac{e^2 n^{1/3}}{4\pi n e^2 \lambda_D^2}$$

$$\Gamma = \frac{1}{4\pi} \frac{1}{\underbrace{(n\lambda_D^3)}_{}^{2/3}}$$

the # of particles in a cube of side λ_D

$\Rightarrow n\lambda_D^3 =$ the # of particles in shielding

- for $\Gamma \ll 1$: $n\lambda_D^3 \gg 1$
- for $\Gamma \gg 1$: $n\lambda_D^3 \ll 1$

Maxwell-Boltzmann Distribution (notation introduction)

In thermal equilibrium (or in many quasi-static processes), the probability of finding a particle with energy E is proportional to $\sim C e^{-(E/T)}$

C = constant

→ Take E to be given by kinetic energy and potential φ

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$E = \tfrac{1}{2} m v^2 + e\varphi$$

energy

for a particle of velocity v in a region of potential φ

Figure 11.2 Different plasma systems indicated on a plot of the number density n of charged particles against the temperature T.