Lecture 8 – Introduction to Gravity 09/22/16

(Schutz ch. 8.1)

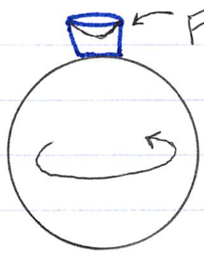
We must postulate a law which shows how the sources of the gravitational field determine the metric – Poisson's equation

the Newtonian analog

$\nabla^2 \phi = 4\pi G \rho$ – density structure determines reaction to gravity $\Rightarrow \phi = -\frac{Gm}{r}$ for a point particle, mass=m

not Lorentz invariant!

→ This equation implies that gravity acts instantaneously!



← Feeling the effects of gravity:

Water sitting at the North Pole in a bucket would rise at the edges, whether there was a still reference frame to prove Earth's rotation or not

→ "Intrinsic inertia"

★A <u>true</u> inertial frame is NOT DEFINABLE with gravity!

(that is, one that is separable from gravity)

Two kinds of Newtonian forces

① $\vec{F}_I = m_I \vec{a}$ m_I = inertial mass

② $\vec{F}_G = m_G \vec{a}$ m_G = gravitational mass

<u>Einstein's Equivalence Principle</u>: $m_I = m_G$

↳ in which Einstein assumed something was right and looked at the consequences ↴

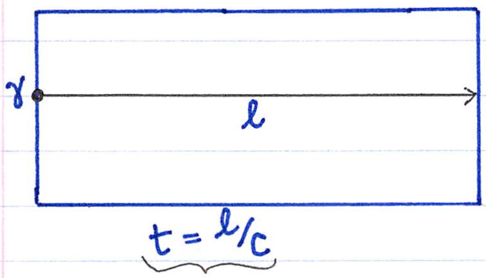
Einstein's observation – the gravitational force as experienced locally while standing on a massive body is actually the same as the pseudo-force experienced by an observer in a non-inertial frame of reference.

→ The frame in which gravity acts <u>is</u> the inertial frame (i.e., the frame in which Special Relativity exists)

Compare the travel of light in the [free-fall] lab frame and the Earth-view frame:

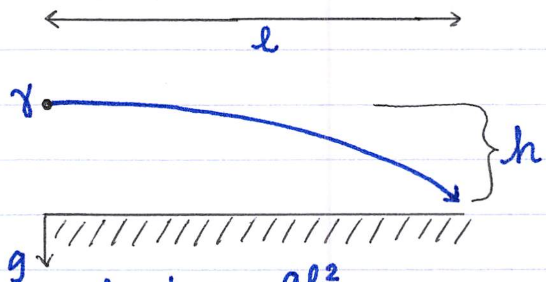
*assuming time is the same (non-relativistic motion in $\hat{y}$) →

free-fall lab: no forces felt



$t = l/c$

Earth View



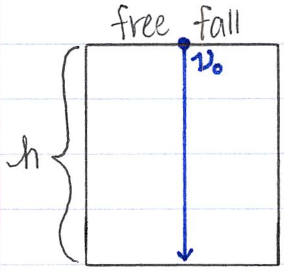
$h = \frac{1}{2} g t^2 = \frac{g l^2}{2c^2}$

plugging in

What about time and gravity?

→ **Gravitational time dilation**

free fall



@$t=0$, let the box freely fall and simultaneously release a pulse of light (frequency $= \nu_0$) from the top of the box.

light propagation time

$t \approx h/c$; $\; v = gt = g(h/c)$

speed of box when light reaches the bottom

Potential energy / unit mass $= \varphi$

Doppler Effect:

$$\nu_1 = \nu_0\left(1 + \frac{v}{c}\right) = \nu_0\left(1 + \frac{gh}{c^2}\right) = \nu_0\left(1 + \frac{\varphi}{c^2}\right)$$

"frame-induced"

Now – **Remove the box** (the box is just an artifact)

↳ **The frequency still shifts! This must be due to gravity!**

Time must be running differently at the two different points (separated by h)

⇒ **Gravity causes changes in frames**

(GR as the manifestation of the frame change when SR is still holding)

<u>Why do masses create gravity?</u>

Start with the Lorentz transformation

→

$$\left.\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ \gamma &= \left[\sqrt{1-\left(\frac{v}{c}\right)^2}\right]^{-1} \end{aligned}\right\} \xrightarrow{\text{compactify}} \left\{\begin{aligned} x^0 &= ct \\ x^1 &= x \\ x^2 &= y \\ x^3 &= z \end{aligned}\right.$$

→ express as a matrix relation

$$x^{\alpha'} = \sum_{\beta=0}^{3} \Lambda^{\alpha'}_{\ \beta}\, x^{\beta}$$

β here is a dummy index

β here $= v/c$

$$\Lambda^{\alpha'}_{\ \beta} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$= \Lambda^{\alpha'}_{\ \beta} x^{\beta}$ (can just drop summation)

To reverse, just change signs on γ-terms

$$x^{\alpha} = \Lambda^{\alpha}_{\ \beta'} x^{\beta'}$$

$$\Lambda^{\alpha}_{\ \beta'} = \begin{pmatrix} -\gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & -\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

such that

$$x^{\alpha} = \underbrace{\Lambda^{\alpha}_{\ \beta'}[\Lambda^{\beta'}_{\ \beta}}_{\text{repeated index means dot product}} x^{\beta}]$$

$$= \delta^{\alpha}_{\beta} x^{\beta} = x^{\alpha}$$

only non-zero for $\beta = \alpha$

Recap with this new notation:

"interval thing"

$$ds^2 = c^2 \underbrace{d\tau^2}_{\tau \text{ as proper time}}$$

$$ds^2 = c^2 dt^2 - dx^2 - \ldots$$
$$= \eta_{\alpha\beta}\, dx^{\alpha} dx^{\beta}$$

→

where

$$\eta_{\alpha\beta} = \begin{array}{l} \quad x_0 = ct \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

such that

$$ds^2 = \eta_{\alpha\beta}\, dx^\alpha dx^\beta = \eta_{00}(cdt)(cdt) + \eta_{11}(dx)(dx) + \eta_{22}(dy)(dy) + \eta_{33}(dz)(dz)$$

Interpreting results:

$$\begin{cases} ds^2 > 0 \rightarrow \text{time-like separations} \\ ds^2 < 0 \rightarrow \text{space-like separations} \\ ds^2 = 0 \rightarrow \text{null-like separation (what photons do)} \end{cases}$$