Lecture 7 - Wrapping Up Special Relativity 09/20/16

Postulates of Special Relativity:

geometry < ① c = constant in all inertial frames of reference
② Laws of physics $\underbrace{\text{the same}}_{\text{covariant}}$ in all inertial frames

↑
BUT! This geometry cannot handle gravity!

Recall: The Lorentz Transformation

$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$
$$\hookrightarrow \gamma = (1 - (v/c)^2)^{-1/2}$$

derived under the condition $\frac{dx}{dt} = c = \frac{dx'}{dt'}$

Then to find $\Sigma F = ma$

$\vec{p} = \gamma m\vec{v}$, our conserved quantity

$$\hookrightarrow F = \frac{d\vec{p}}{dt} = \underbrace{\gamma^3}_{\text{one for each dimension}} m\vec{a} \quad , \quad \vec{a} = \frac{d^2\vec{x}}{dt^2}$$

Energy:

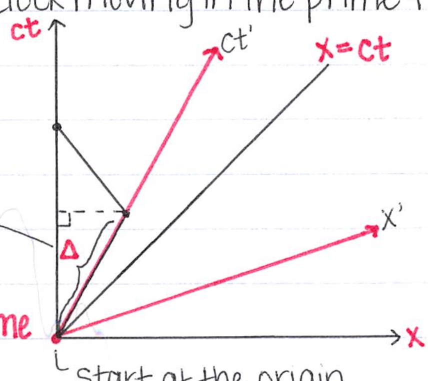
$E = mc^2$ only applicable in the mass' rest frame

$$E = \gamma mc^2 \Rightarrow KE = \underbrace{\gamma mc^2}_{\text{total E}} - \underbrace{mc^2}_{\text{rest E}}$$

Time Dilation:

ex. for a clock moving in the prime frame

one interval in the prime frame



start at the origin

↑
(!!) The clock has not moved in its own frame!

$dx' = 0$

KNOWN:

$dt' = \Delta$, $dx' = 0$, $dx = v\,dt$

↪ applying lorentz equation ↓

$$dx' = \gamma(dx - v\,dt)$$
$$dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right)$$

⇑
Want to find this!

→

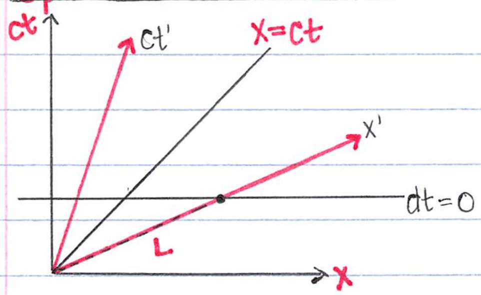
plugging in for dx

$$dt' = \gamma\left(dt - \frac{v^2 dt}{c^2}\right)$$

$$= \gamma dt \underbrace{\left(1 - \frac{v^2}{c^2}\right)}_{1/\gamma^2} = \underbrace{\frac{dt}{\gamma} = dt'}$$

CAUTION! This is only true for our specific conditions!

Space Contraction:



KNOWN:

$dx' = L$ – the proper length

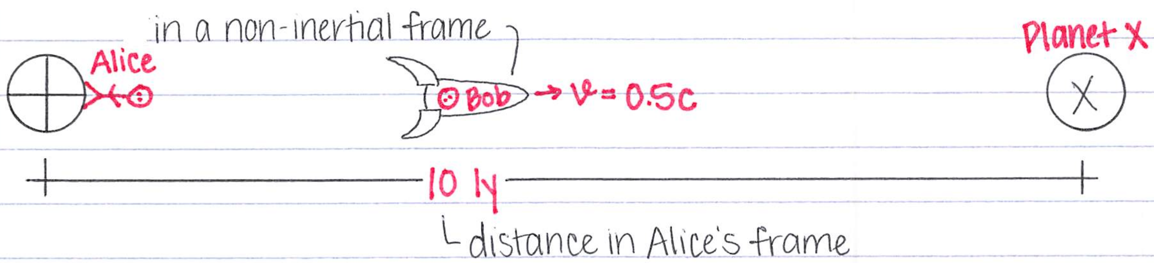
$dt = 0$ ⇒ What it means to be measuring a length in an instant in time in the un-primed frame

$$dt' = \gamma(\cancel{dt}^{\,0} - v\,dx/c^2)$$

$$= \underbrace{-\gamma \frac{v\,dx}{c^2}}$$ ⇒ $dx = \dfrac{dx'}{\gamma}$

An instant in time in the un-primed frame is NOT an instant in time according to the prime observer!

The Twin Paradox



Alice and Bob are 20 year-old twins at the time that Bob leaves for Planet X.

Be careful! Bob is not an inertial observer!

↳ use Alice as observer

→

Time according to Alice:

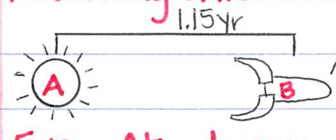
$\Delta t_A = \frac{20\ \text{ly}}{0.5c} = 40$ years

$\Delta t_B = \frac{\Delta t_A}{\gamma} \cong 34.6$ years → less time has passed for Bob!

"the moving clock runs slow"

⇒ **Lack of Global Simultaneity** ⇐

Resolving this:



Bob is counting the light pulses from Alice

$\Delta t_B = \Delta t_A \gamma \cong \underbrace{1.15 \text{ years}}$

but this wouldn't yield 40 flashes in the shorter total time above

Every $\Delta t_A = 1$ year, Alice sends out a light pulse

* **Light Travel Effect** *

└ it takes extra time for the light to "catch up" to Bob

$\Delta t_{flash} = 1.15\text{ yr} + \underbrace{\left[\frac{(0.5c)(1.15\text{ yr})}{c}\right]}_{\text{accounts for light travel time}} \cong 1.73$ years between flashes

Compute the # of flashes seen on outgoing trip

$N_{flash\ out} = \left(\frac{1\text{ flash}}{1.73\text{ yr}}\right)\left(\frac{34.6\text{ yr}}{2}\right) = 10$ flashes

└ half of the trip according to Bob

Then on the way back

↳ Bob "catching up" to light now ↴

$\Delta t_{flash} = 1.15\text{ yr} - \left[\frac{(0.5c)(1.15\text{ yr})}{c}\right] \cong 0.577$ year between flashes

$N_{flash\ in} = \left(\frac{1\text{ flash}}{0.577\text{ yr}}\right)(17.3\text{ yr}) = \underbrace{30 \text{ flashes}}$

We've "caught up" and experienced all 40 flashes we know were sent out by Alice, even though Bob only experiences 34.6 years!

***Cole's alternate explanation:** On the way back, if the traveling twin emits a light pulse, the Earth-bound twin does not receive it until the traveling twin is almost home. And until that pulse is received, the Earth twin thinks the traveling clock is going slower. This asymmetry explains why the traveling twin ages less.

⇒ When the ship turns around, until Earth receives that pulse, one twin has changed frames while the other has not