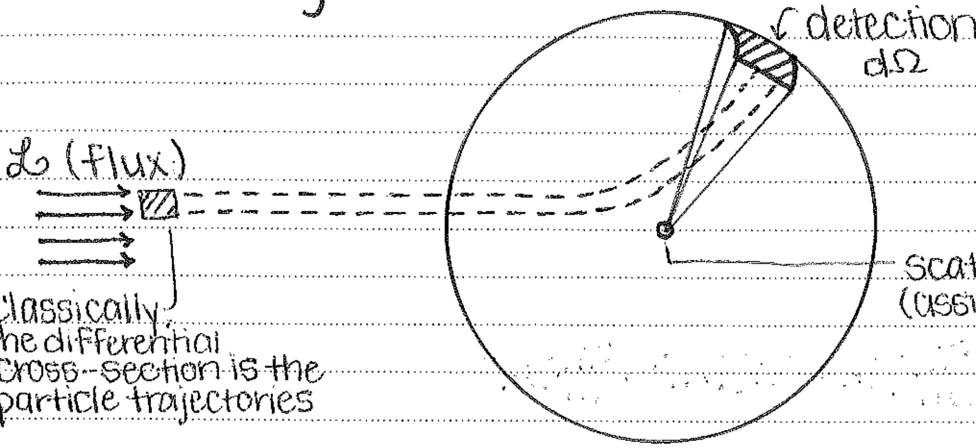
11/30/12

Scattering



Classically, the differential cross-section is the particle trajectories

$$\frac{1}{\mathcal{L}}\frac{dN}{d\Omega} = \frac{d\sigma}{d\Omega} = \text{differential cross-section} \quad \text{(units of area)}$$

dN: # particles/time

$\mathcal{L}$: Luminosity (# particles/time/area)

* In quantum scattering, the differential cross-section cannot describe a specific area because the particles don't have trajectories, they come from all over with a certain probability

→ probability density

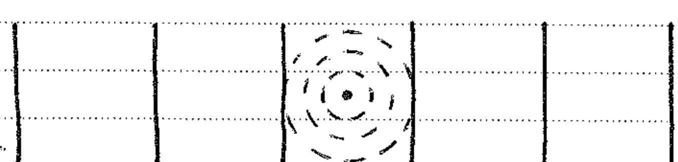
$$\text{total cross-section} = \int d\Omega \frac{d\sigma}{d\Omega}$$

For quantum scattering, find a solution to the TISE that takes the following form:

$$\psi(r,\theta) \xrightarrow[r\to\infty]{} \overset{\text{normalization}}{A}\Big[\underbrace{e^{ikz}}_{\text{incoming plane wave}} + \underbrace{f(\theta)\frac{e^{ikr}}{r}}_{\text{outgoing spherical wave}}\Big]$$

$f(\theta)$ - scattering amplitude (complex number)

this equation only valid for large distances (need TDSE near scattering center)



incoming plane wave — outgoing spherical wave

→

Aside: Probability Density

$$\mathbf{j} = \frac{-i\hbar}{2m}(\psi^* \nabla \psi - \nabla \psi^* \psi)$$

$\boxed{\hat{n}}$ hole in space

$$\frac{d}{dt}|\psi|^2 + \nabla \cdot \mathbf{j} = 0$$

#particles going through hole $= \int da\, \mathbf{j} \cdot \hat{n}$

*every point in space has a different probability flow

$$\underbrace{\mathcal{L}}_{|\mathbf{j}|} = \underset{\text{flux}}{\text{incoming}} = \frac{-i\hbar}{2m}(\overbrace{ik}^{\psi^*\nabla\psi} + \underbrace{ik}_{-\nabla\psi^*\psi})|A|^2 = \frac{\hbar k}{m}|A|^2$$

$$\frac{d\sigma}{dN} = \frac{1}{\mathcal{L}}\frac{dN}{d\Omega} = \underbrace{\frac{m}{\hbar k |A|^2}}_{1/\mathcal{L}} \underbrace{\left(|f|^2 \frac{-i\hbar}{2m}(ik + ik)\frac{|A|^2}{r^2}\right)}_{\mathbf{j}} \underbrace{r^2 d\Omega}_{\text{area of the hole}} \overbrace{\frac{1}{d\Omega}}^{\text{size of the hole (radians)}}$$

$$= |f(\theta)|^2$$

$\hookrightarrow Af \frac{d}{dr}\frac{e^{ikr}}{r} = Af\left[\frac{ik}{r} - \cancel{\frac{1}{r^2}}\right]e^{ikr}$ negligible for large r

Born Approximation: small potential ($f(\theta)$ and e^{ikr}/r are very small)

$\rightarrow$ finding $f(\theta)$ using perturbation theory

$$\psi(\mathbf{r}) = \underset{\text{incoming}}{\psi_{in}} + \underset{\text{outgoing}}{\psi_{refl}}$$

Satisfies: $\frac{-\hbar^2}{2m}\nabla^2(\cancel{\psi_{in}} + \psi_{refl}) + \underbrace{V(\psi_{in} + \psi_{refl})}_{\simeq V\psi_{in}} = E(\cancel{\psi_{in}} + \psi_{refl})$

(small thing)^2

$$\underbrace{\left[\nabla_r^2 + \frac{2mE}{\hbar^2}\right]}_{\text{linear operator}} \underbrace{\psi_{refl}(\mathbf{r})}_{\text{want to know}} = -\left(\frac{2m}{\hbar^2}\right)\underbrace{V(\mathbf{r})\psi_{in}(\mathbf{r})}_{\text{known}}$$

$$Ax = b$$

$$\underbrace{A^{-1}A}_{\mathbb{I}}x = A^{-1}b$$

works for any b

Inverse of operator $\Rightarrow$ Greene's function

"$AA^{-1} = \mathbb{I}$"

$$\left[\nabla_r^2 + \underbrace{\frac{2mE}{\hbar^2}}_{k^2}\right]G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$$

note that it's independent of potential! "any b"

"$x = A^{-1}b$"

$$\Rightarrow \psi_{refl}(\mathbf{r}) = -(2m/\hbar^2)\int d^3\mathbf{r}'\, G(\mathbf{r}, \mathbf{r}')V(\mathbf{r}')\psi_{in}(\mathbf{r}')$$

only thing on $\mathbf{r}$ $\rightarrow$ operator forms δ-function to create $V(\mathbf{r})$

$$k = \sqrt{2mE}/\hbar$$

$$G(\mathbf{r}, \mathbf{r}') = \frac{-1}{4\pi}\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

Coulomb Law