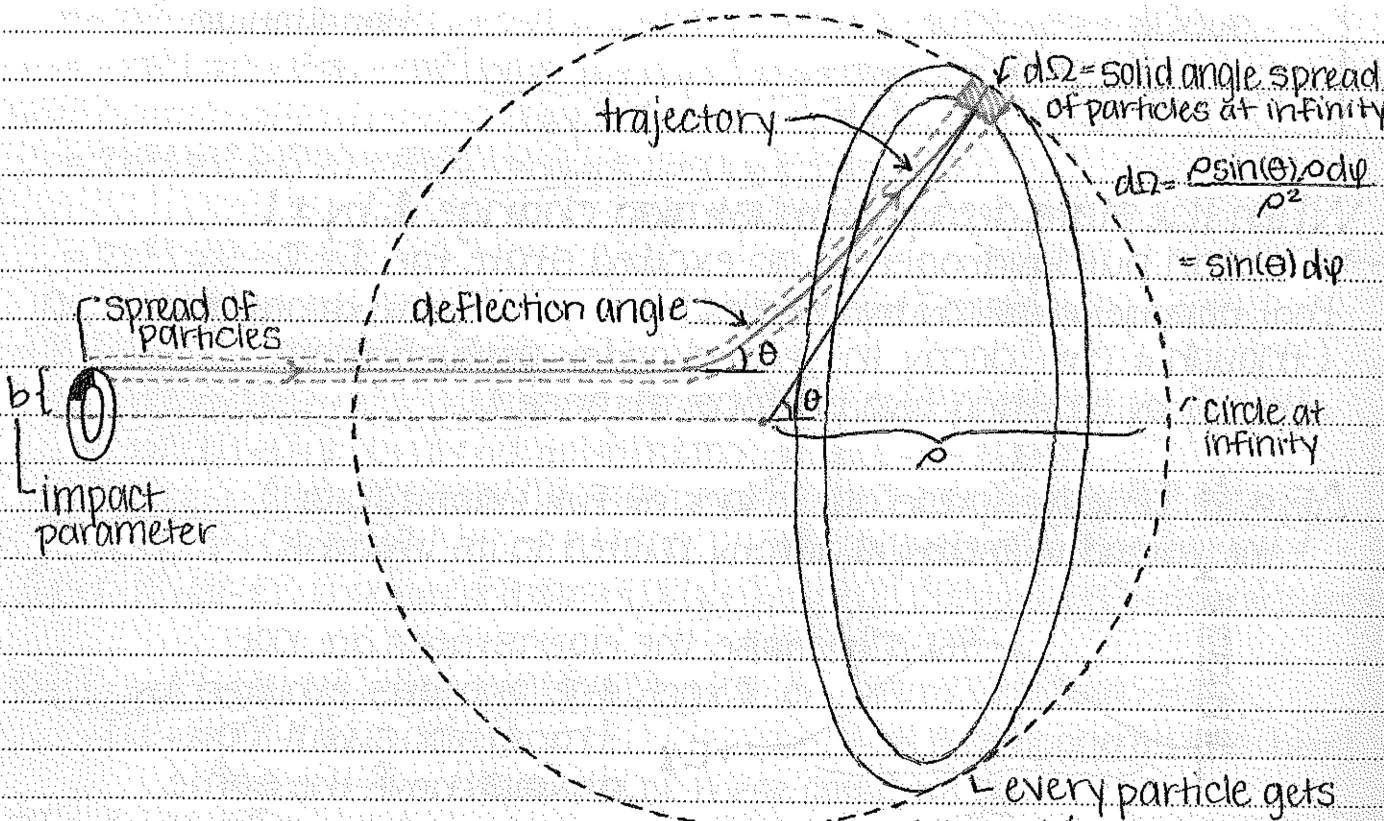
Scattering

"Do you know what scattering is? You throw things at each other and figure them out! Want to know how a clock works? Throw it at your friend, gears come out! And you're like, "Oh! That's how it works!"

Classical Scattering:

"Remember, in classical physics, particles have both position and momentum at the same time! How weird!"



every particle gets deflected (some more, some less, but all move)

differential cross-section $= \left|\frac{d\sigma}{d\Omega}\right|$ (units of area)

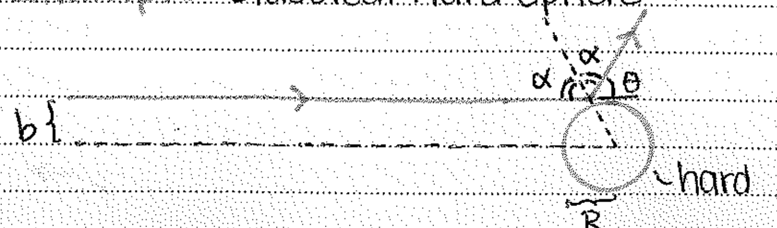
cross-section always positive

total cross-section $= \int d\Omega \left|\frac{d\sigma}{d\Omega}\right| = \sigma$

only way this does not blow up to infinity is if your potential has a finite range ("and this is why scattering in classical mech. is really just stupid")

Example: Classical Hard Sphere

"examples are good for the soul"



$2\alpha + \theta = \pi$

$b = R\sin(\alpha) = R\sin(\frac{\pi}{2} - \frac{\theta}{2}) = R\cos(\frac{\theta}{2})$

$$\frac{d\sigma}{d\Omega} = \left| \frac{b\,db\,\cancel{d\varphi}}{\sin(\theta)\,\cancel{d\varphi}\,d\theta} \right| = \frac{b}{\sin(\theta)}\frac{db}{d\theta} = \frac{R\cos(\theta/2)}{\sin(\theta)}\frac{R\sin(\theta/2)}{2} = \frac{R^2}{4}$$

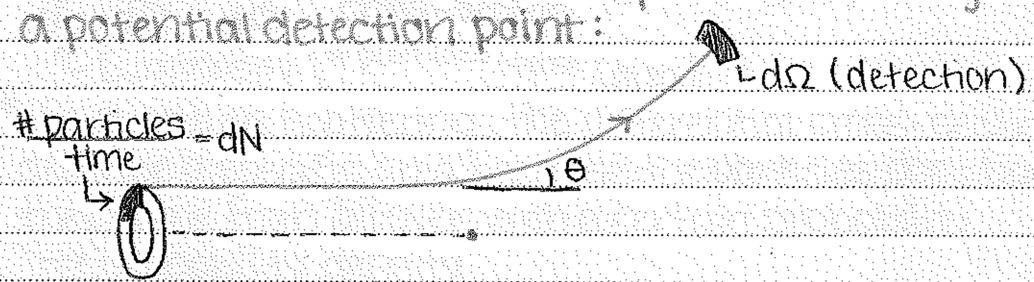
needed this to go away because it gets deformed from scattering

$$\sigma = \underbrace{\int d\Omega}_{4\pi} \frac{R^2}{4} = \pi R^2$$

exactly as expected

"This is true for any hard shape: a hard square, a hard pony.... in the spirit of quantum physics, use a cat and shoot it with water..."

Now assume an infinite flux of particles moving towards a potential detection point:



Luminosity (# particles/area/time)

$$dN = \mathcal{L}\,\underbrace{b\,db\,d\varphi}_{\sigma = \text{area}}$$

how many particles of this infinite flux that pass through this given cross-section (this gets rid of our impact parameter, which is hard to find in classical mechanics and impossible in Quantum Mechanics)

$$\text{differential cross-section} = \frac{1}{\mathcal{L}}\frac{dN}{d\Omega} = \frac{d\sigma}{d\Omega}$$

Quantum scattering:

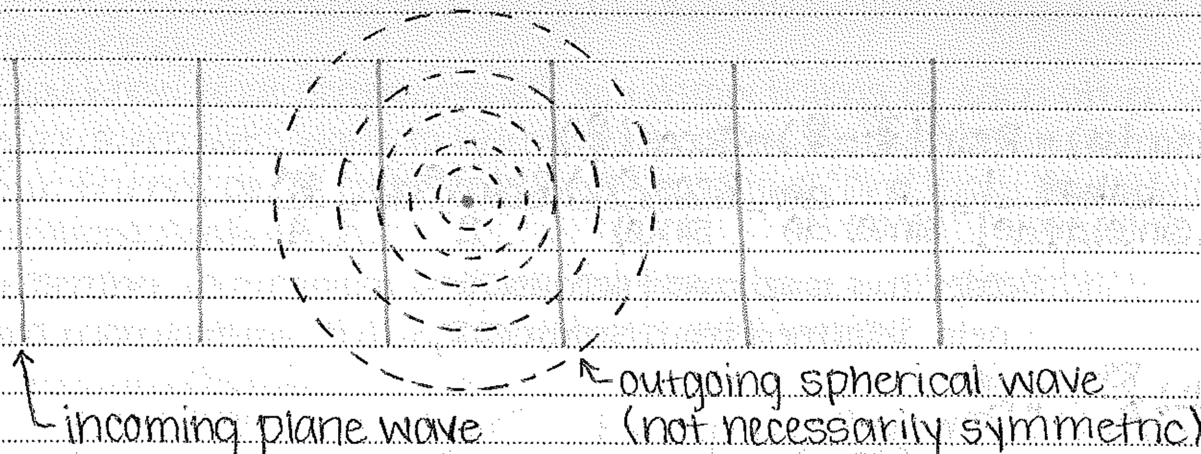
→ must make a "wave pocket" to compromise between position and momentum

Goal - find a solution to the time-independent Schrödinger equation that takes this form:

arbitrary normalization — plane wave moving in the z-direction towards the scattering center

$$\psi(r,\theta) \longrightarrow A\left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right]$$

no φ dependence (for simplicity) — outgoing spherical wave


incoming plane wave
outgoing spherical wave
(not necessarily symmetric)