QM- Lecture 9

February 25^{th} 2016

* HW Problem: Write $2P_{1/2}, F=0$

in terms of $|1 m_\ell\rangle |1/2\, m_s\rangle |1/2\, m_I\rangle$ states

$$1 \otimes (1/2 \otimes 1/2)$$

$$1 \otimes (1 \oplus 0)$$

$$(1 \otimes 1) \oplus (1 \otimes 0) \rightarrow 2 \oplus 1 \oplus 0 \oplus 1$$

4 diff irr. representations

A Note About Faster Ways.... $\Downarrow$ efficiency...

$\vec{F}|\psi\rangle = 0 \longleftrightarrow$ singlet

$F_z|\psi\rangle = 0 \Rightarrow$ this imposes $m_\ell + m_s + m_I = 0$

$F_\pm|\psi\rangle = 0$

EXAMPLE: $1/2 \otimes 1/2 = 1 \oplus 0$

$$(1/2 \otimes 1/2) \otimes 1/2 = (1 \oplus 0) \otimes 1/2 = (1 \otimes 1/2) \oplus 1/2$$
$$= 3/2 \oplus 1/2 \oplus 1/2$$

$$1/2 \otimes 1/2 \otimes 1/2 \otimes 1/2 = (1 \oplus 0) \otimes (1 \oplus 0) = 2 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0$$

* Note on hyperfine: ${}^{133}Cs$, $I = 7/2$; $\vec{F} = \vec{J} + \vec{I}$

$J = 1/2$

$$F = \frac{1}{2} \otimes \frac{7}{2} = 4 \oplus 3$$

transition between these two defines units of a second

9,192,631,770 periods of the $F=3 \rightarrow F=4$ transition freq = 1 sec.

EXAM: Last class before Spring Break

1 more HW to be assigned

> proved: $\langle \omega' j' m' | \vec{V}_1 | \omega j m \rangle = \delta(\omega' j' \omega j) \langle \omega' j' m' | \vec{V}_2 | \omega j m \rangle$
if $\vec{V}_{1,2}$ are vector ops.

> Used this to prove projection theorem:

$$\underbrace{\langle \omega' j m' | \vec{V} | \omega j m \rangle}_{\text{same}} = \frac{1}{j(j+1)} \langle \omega' j m' | (\vec{V} \cdot \vec{J}) \vec{J} | \omega j m \rangle$$

Used Projection Theorem to diagonalize $\langle n j l m_j' | S_z | n j l m_j \rangle$

> Extended to tensor operators. T_{kq} (k: j-values, q: J_z-values)

EX: $Y_{lm}(\theta, \phi)$; electric quadrupole moment $Q^{ij} = \int \rho (x^i x^j - \frac{1}{3} x^2 \delta_{ij}) \, d^3x$

$$\sum_j Q^{ij} = 0$$

* symmetry preserved under rotation
* trace-less-ness preserved

Q^{ij} (3, 3 values)

→ 5 independent elements (w/ traceless thing imposed)

→ consistent w/ $j=2$, $2j+1 = \underline{5}$

SELECTION RULES

$\langle \omega' j' m' | T_{kq} | \omega j m \rangle$
vanishes unless

1) $m' = q + m$
2) $j' \subset k \otimes j$
this symbol....?
[contained in] ✓

* what it means to be a tensor op...
$[J_z, T_{kq}] = q\hbar T_{kq}$
$[J_\pm, T_{kq}] = \sqrt{k(k+1) - q(q\pm1)}\, \hbar \, T_{k,q\pm1}$

* $T_{kk} | jj \rangle$ is the "top" state, largest J_z

WIGNER-ECKART THEOREM

$$\langle \omega' j' m' | T_{kq} | \omega j m \rangle = \left(\frac{\langle \omega' j' \| T_k \| \omega j \rangle}{2j+1} \right) \underbrace{\langle j' m' | k j q m \rangle}_{\text{Clebsch-Gordon Coefficients}}$$

EXAMPLE: SPRING 2016 QUAL PROBLEM
[QUANTUM, #4?]

a) $\langle n' \overset{\ell=3}{\ell'} m | Y_{30} | \overset{\ell=4}{1\,4\,0} \rangle$

1) 1st selection rule : $m = 0 + 0 = 0$ } otherwise vanishes
2) 2nd selection rule : $\ell' = 1, 2, 3, 4, 5, 6, 7$ } otherwise vanishes
3) Parity $|140\rangle$ even parity
Y_{30} odd parity
$\Rightarrow$ zero unless ℓ' is odd

b) $\langle n \ell m | Y_{40} | n \ell m \rangle$

1) ℓ must lie between $|\ell - 4|$ to $|\ell + 4|$
vanishes if $\underset{4}{\ell = 0}$, $\underset{3,4,5}{\ell = 1}$, { OKAY for $\ell = 2$, + up }

* nothing from parity or 1st rule

c) $\langle n \ell m | \underline{Y_{30}} | n \ell m \rangle$

1) PARITY Y_{30} odd.... so $Y_{30} | n \ell m \rangle$ has opp. parity $\Rightarrow$ ALL VANISH

d) $\langle 220 | \underbrace{\lambda V(r) Y_{20}}_{\text{irreducible tensor opp}} | 220 \rangle = \varepsilon = A \langle 20 | 2200 \rangle$ } by Wigner Eckhart Thm

Consider $\langle 222 | \lambda V(r) Y_{20} | 222 \rangle = A \langle 22 | 2202 \rangle$ } by Wigner Eckhart Thm