QM-Lecture 8

Zeeman Effect

February 23rd 2016

$H_Z = -\vec{\mu}\cdot\vec{B}$ where for an electron, $\vec{\mu} = \frac{e}{2m_ec}(\vec{L} + \underset{\uparrow 2}{g_e}\vec{S})$

if $B < 10^5$ gauss $= 10\,T$: then Zeeman effect is small compared to fine structure; treat H_Z as perturbation, unperturbed states being $|njlm_j\rangle$ {these are degenerate w/ E_{nj}}

Diagonal? ⇒ $\frac{-eB}{2m_ec}\{L_z + g_eS_z\}$ in Schwabl, $= J_z + (g_e - 1)S_z$

* J_z is diagonal in $|njlm_j\rangle$.

* S_z is <u>NOT</u>

↳ need to diagonalize ⇒ $\langle njlm_j'|S_z|njlm_j\rangle$

<u>However</u>: $[J_z, S_z] = 0$

using this in our matrix element, it implies that it's zero unless $m_j = m_j'$

* Just need to find expectation value

$\langle njlm_j|S_z|\underbrace{njlm_j}\rangle$

these arise from: $l \otimes 1/2 = (l - 1/2) \oplus (l + 1/2)$

$= \frac{\pm\hbar m_j}{2l+1}$ (for $j = l \pm 1/2$) {evaluated earlier in Schwabl}

Deal With All This Now w/ Vector/Tensor Operators...

VECTOR OPERATORS

V^a ; $a = 1, 2, 3$ ⇒ $[\underset{\text{generator of Rotations}}{J^a}, V^b] = i\varepsilon^{abc}V^c$ this defines a vector operator

ex. of V^a... $\vec{X}$, $\vec{p}$, $v(r)\vec{S} + w(r)\vec{L}$

herical Vectors, $Y_{l=1,\, m=-1,0,1}$ {these are combinations of the V^a's}

Y_{10} | $\propto \cos\theta \longleftrightarrow z$

Y_{11} | $\propto -\sin\theta\, e^{i\phi} \longleftrightarrow -(x+iy)$ $\Longrightarrow$

$V^a \leftrightarrow V_{1m}$

$V_{10} = V_z$

$V_{1,\pm1} = \mp(V_x \pm iV_y)$

Y_{1-1} | $\propto \sin\theta\, e^{-i\phi} \longleftrightarrow x-iy$

* Consider $J_z,\ J_\pm = J_x \pm iJ_y$

↳ $[J_z, V_{1m}] = m\hbar V_{1m}$

↳ $[J_\pm, V_{1m}] = \sqrt{2}\hbar V_{1,m\pm1}$

} understand that $V_{1m} = 0$ if $|m| > 1$

↳ $[L_\pm, Y_{1m}] = \sqrt{2}\hbar\, Y_{1,m\pm1}$ =

Amazing Fact: if $\vec{V}_1$ and $\vec{V}_2$ are V-ops.

then....

$$\langle \omega' j' m' | \vec{V}_1 | \omega j m \rangle \underset{\substack{\uparrow \\ \text{proportional}}}{\propto} \langle \omega' j' m' | \vec{V}_2 | \omega j m \rangle$$

* the constant relates them

only depends on $\omega', j', \omega, j \Rightarrow \underset{[\text{constant}]}{\delta(\omega', j', \omega, j)}$

extremely [illegible] important special case: If we take the V's to be J's...

the j, j' must be the same to have a nonzero matrix element

↳ $$\langle | V | \rangle = \delta \langle \omega' \underbrace{j\, m' | \vec{J} | \omega j}_{\text{same}}\, m \rangle$$

Find the constant δ in the case $\vec{V}_2 = \vec{J}$:

$$\langle \omega' j m' | \vec{V} | \omega j m \rangle = \delta_{\omega\omega j} \langle \omega' j m' | \vec{J} | \omega j m \rangle$$

→ Dot this equation w/ $\vec{J}$ ... insert some comp. of $\vec{J}$ ... all it does it create linear combos of states that still works, since valid for all m's.

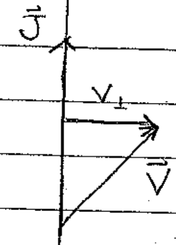
$$\langle \omega' j m' | \vec{V}\cdot\vec{J} | \omega j m \rangle = \delta \langle \omega' j m' | J^2 | \omega j m \rangle$$
$$= \delta\, j(j+1)\hbar^2$$

SO: $\delta = \dfrac{\langle \omega' j m' | \vec{V}\cdot\vec{J} | \omega j m \rangle}{j(j+1)\hbar}$

actually product of 2 elements
⇑
[compact form]

Implies PROJECTION THEOREM: $\langle \omega', j, m' | \vec{V} | \omega j m \rangle = \dfrac{\langle \omega' j m' | (\vec{V}\cdot\vec{J})\vec{J} | \omega j m \rangle}{j(j+1)\hbar^2}$

* $\dfrac{(\vec{V}\cdot\vec{J})\vec{J}}{j(j+1)\hbar^2}$: component of $\vec{V}$ in $\vec{J}$ direction



$\langle | \vec{V}_\perp | \rangle = 0$

☆ NOW: Use all this in the Zeeman effect (as an example)

Consider $\langle n j l m_j | \overset{S_z}{\vec{S}} | n j l m_j \rangle$ ... by projection thm

$= \dfrac{\langle n j l m_j | (\vec{S}\cdot\vec{J})\overset{J_z}{\vec{J}} | n j l m_j \rangle}{j(j+1)\hbar^2}$ Choose $\vec{J} = J_z$ in our case since for Zeeman, S_z

$= \dfrac{m_j \hbar}{j(j+1)\hbar^2} \langle n j l m_j | \vec{S}\cdot\vec{J} | n j l m_j \rangle$ → $\vec{J} - \vec{S} = \vec{L}$

$J^2 + S^2 - 2\vec{S}\cdot\vec{J} = L^2$

$\vec{S}\cdot\vec{J} = 1/2\,(J^2 + S^2 - L^2)$

$= \dfrac{\hbar^2}{2}\left(j(j+1) + s(s+1) - l(l+1)\right)$

$(s = 1/2)$

Consider this element...

spherical vector, $q = 1, 0, -1$

$$\langle \underbrace{\omega' j'}_{\text{fix}} m' | \underset{\text{vary } q}{V_{1q}} | \underbrace{\omega j}_{\text{fix}} m \rangle$$

$\boxed{k=1}$

$[J_z, V_{1q}] = q V_{1q}$ Take expectation value...

$$\langle | J_z V_{1q} | \rangle - \langle | V_{1q} J_z | \rangle$$
$$= m' \langle | V_{1q} | \rangle - m \langle | V_{1q} | \rangle = q \langle | N_{1q} | \rangle$$

Either $\langle | V_{1q} | \rangle = 0$, OR $\underline{\underline{q = m' - m}}$ example of selection rule.

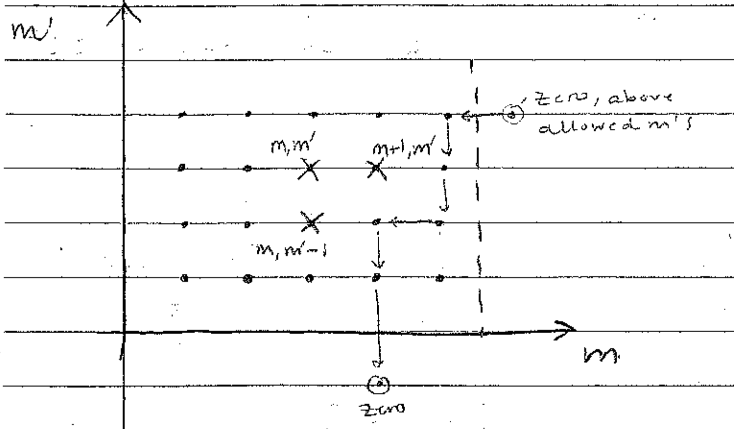
Selection Rule
$\Rightarrow m' = q + m$

All components of $\langle m' | V_{1q} | m \rangle$ are determined from any one of them by the commutator relation:

$$[J_+, V_{1q}] = \sqrt{2}\, V_{1q+1}$$

Taking $\langle m' | \ | m \rangle$...

$$\sqrt{2} \langle m' | V_{1q+1} | m \rangle = \langle m' | J_+ V_{1q} | m \rangle - \langle m' | V_{1q} J_+ | m \rangle$$
$$= \sqrt{2} \langle m'-1 | V_{1q} | m \rangle - \sqrt{2} \langle m' | V_{1q} | m+1 \rangle$$



all can be determined...

[things got super weird here...]

"one comp. determines all of them"

unrestrict $\ell = 1$.... $T_{kq} \Rightarrow Y_{\ell m}$

$\Rightarrow [L_z, Y_{\ell m}] = m\hbar Y_{\ell m}$

$[L_\pm, Y_{\ell m}] = \sqrt{\ell(\ell+1) - m(m\pm1)}\, \hbar\, Y_{\ell m \pm 1}$

{Spherical Tensor Operators}

[notes posted explaining this?]