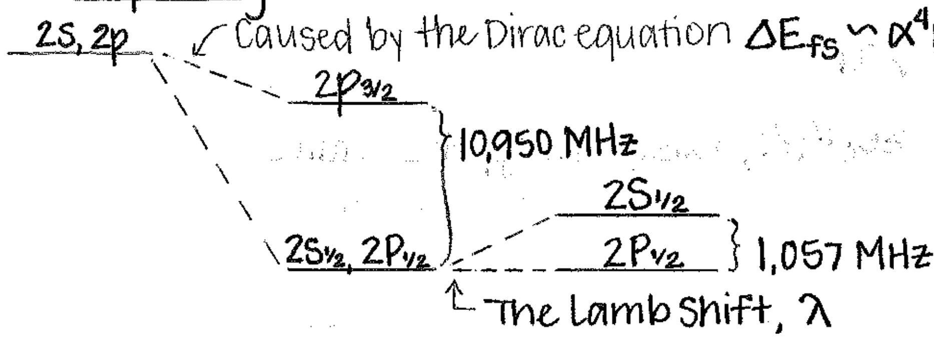
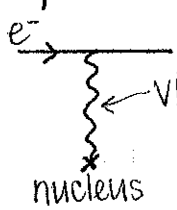
Lecture 6 – Fine Structure Corrections 02/16/16

1. Level splitting
2. Hyperfine splitting

1. Level Splitting

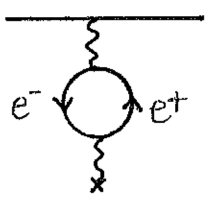


Feynman Diagrams: λ-shift



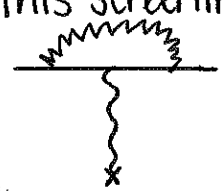
← virtual photon; gauge boson of the electromagnetic interaction

this electron/photon interaction can be described as Vacuum Polarization



→ the photon splits into an e^-/e^+ pair then recombines
→ this process/phenomenon screens the field of the nucleus ← closer to the nucleus ⇒ stronger screening (field screening actually infinite)

This screening causes a sort of "self-energy"
also actually infinite



→ the self-energy shifts the quantized energy levels of the electron by adding extra jiggling.

*We must use renormalization theory to quantify these values (i.e., see them as non-infinite)

extra jiggle $= \delta r^i$

$\overline{\delta r^i} = 0$
↖ average jiggle
jiggle in 2D

but the (average jiggle)$^2 \neq 0$

$\overline{\delta r^i \delta r^j} = \frac{1}{3}\delta^{ij}$

→ $\overline{\delta r^2}$ is the size of the extra jiggle

<u>Energy of the jiggle:</u>

$$U(\vec{r}+\delta\vec{r}) = U(\vec{r}) + \tfrac{1}{2}(\partial_i\partial_j U)\underbrace{\overline{\delta r^i \delta r^j}}_{\frac{1}{3}\delta^{ij}\overline{\delta r^2}} + \dots \quad \text{(Taylor expansion)}$$

$$= U(\vec{r}) + \frac{\overline{\delta r^2}}{6}\underbrace{\nabla^2 U}_{Ze_0\delta^3(\vec{r})}$$

$Ze_0\delta^3(\vec{r})$, raises energy of S-states
(and only at the origin)

$\Delta E_{lamb} \sim \alpha^5 mc^2$

*note: vacuum polarization lowers the energy of S-states

2. Hyperfine Splitting/Structure

We must consider the dipole moment of the H-atom

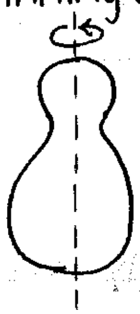
Nuclear magnetic moment – about a factor of 2,000 smaller than the electronic magnetic moment

$$\vec{M} = \frac{Ze_0 g_N}{2M_N c} = \vec{I}$$

$\vec{I}$: total angular momentum of the nucleus

M_N: mass of the nucleus

Suppose you have any axes-symmetric volume [LIKE A PEANUT!] spinning on the axis:



angular momentum

$$\vec{L} = \int \vec{r}\times\vec{p}\, dV$$

$\vec{p} = \rho_m \vec{v}$; momentum is mass-density dependent

electronic magnetic moment

$$\vec{\mu} = \frac{1}{c}\int \vec{r}\times\vec{j}\, dV$$

$\vec{j} = \rho_e \vec{v}$; current is charge-density dependent

comparing the two,

$$\vec{\mu} = \frac{1}{c}\left(\frac{\rho_e}{\rho_m}\right)\vec{J}$$

└ the g-factor, which accounts for the fact that e.g., an electron, does not have $\vec{\mu} \propto \frac{e}{m}\vec{J}$ but actually $\propto 2\vec{J}$

$$\Rightarrow \mu_e = \frac{-e_0 \cdot 2}{2m_e c}\frac{\hbar}{2}$$

How does this $\vec{M}$ affect the energy levels?

$\underbrace{\vec{M}_N \rightarrow \vec{B}_N,}_{\text{turns into a little magnet}}$

this leads to a hyperfine splitting of:

$\rightarrow H_{hyp} = \frac{e_0}{mc} \underbrace{\vec{S} \cdot \vec{B}_N}$

dipole moment (spin interaction with the generated magnetic field)

proton magnetic moment μ_p electron magnetic moment μ_e

For the correction on the spatial part of the wavefunction, (use perturbation theory)

$$\langle n, j=\tfrac{1}{2}, \ell=0 | H_{hyp} | n, j=\tfrac{1}{2}, \ell=0 \rangle = \frac{4}{3} g_N \left(\frac{m}{M_N}\right) (Z\alpha)^4 mc^2 \frac{1}{n^3} \frac{\vec{S} \cdot \vec{I}}{\hbar^2}$$

must diagonalize this term

"small mass" term; the electron-proton mass ratio

- to diagonalize $\vec{S} \cdot \vec{I}$, we must first calculate

total angular momentum of the atom

electronic angular momentum

$F = \vec{S} + \vec{I}$ – nuclear angular momentum

$F^2 = (\vec{S} + \vec{I})^2 = \underbrace{S^2}_{\frac{3}{4}\hbar^2} + \underbrace{I^2}_{\frac{3}{4}\hbar^2} + 2\vec{S} \cdot \vec{I}$

$\frac{\vec{S} \cdot \vec{I}}{\hbar^2} = \frac{1}{2}\left(\frac{F^2}{\hbar^2} - \frac{3}{2}\right)$; $\frac{F^2}{\hbar^2} \leftrightarrow F(F+1)$

hyperfine splitting is sized according to this value of F

$\Delta E_{hyp} \sim \frac{m}{M} \alpha^5 mc^2$