QM-lecture 10

rc. 1st 2016

XAM #1: Next Thurs. ; Bring questions Tuesday?
[Why must we have an exam...] → Post of Piazza?

HW Problem: $\int -\psi^* \nabla^2 \psi = \int -\vec{\nabla}\cdot(\psi^* \vec{\nabla}\psi) + \vec{\nabla}\psi^* \cdot \vec{\nabla}\psi$

{went over Lamb's exp.}

THE VARIATIONAL METHOD

If H has a ground state $H|0\rangle = E_0|0\rangle$
then $\langle\psi|H|\psi\rangle \geq E_0$ for any $|\psi\rangle$

Proof: $|\psi\rangle = a|0\rangle + \sqrt{1-a^2}\,\underbrace{|0_\perp\rangle}_{\text{orthog. state // not neccessarily eigenstate}}$

$\langle\psi|H|\psi\rangle = |a|^2 E_0 + \left(a\sqrt{1-a^2}\,\langle 0|H|0_\perp\rangle + \text{complex conj.}\right)$ zero since orthog.

$\boxed{\text{wrong....}}$ $\quad + (1-a^2)\langle 0_\perp|H|0_\perp\rangle$

DO OVER: $|\psi\rangle = \sum_n c_n|n\rangle$

$\langle\psi|H|\psi\rangle = \sum_{n n'} c_n^* c_{n'} \langle n|H|n'\rangle$

$= \sum_n |c_n|^2 E_n \geq E_0$

Apparently that's it....

IDENTICAL PARTICLES

Spin-statistics Theorem of Relativistic QFT (in ≥ 3 spatial dim).

→ ½ integer spin ⟷ fermions ⟷ antisymmetric states under particle exchange.

→ integer spin ⟷ bosons ⟷ symmetric states

$\left.\begin{matrix} e, \mu, \tau \\ \nu_e, \nu_\mu, \nu_\tau \\ \text{quarks} \end{matrix}\right\}$ fermions, ½ spin $\left.\begin{matrix} \text{photon} \\ W^{\pm}, Z \text{ bosons} \\ \text{Gluons} \end{matrix}\right\}$ bosons, spin-1

Higgs boson → spin-0

Graviton → spin-2

Pauli Exclusion Principle

* Responsible for the stability of matter,
  - → periodic table,
  - → structure of nuclei,
  - → white dwarf neutron stars,
  - → Condensed Matter,

$\Psi(1, 2, \ldots, N)$

↖ particle label

P_{ij} → permutation of particles i ; j

$$P_{ij}\,\Psi(\ldots i \ldots j \ldots) = \Psi(\ldots j \ldots i \ldots) \begin{cases} -\Psi(\ldots i \ldots j \ldots) & \text{fermions} \\ +\Psi(\ldots i \ldots j \ldots) & \text{bosons} \end{cases}$$

* even # of fermions is a boson

↳ H-atm: p ; e → even # of fermions, so a boson.

Ground State of Helium Atom (neglect e-e interaction)

2s, 2p ____

1s ● ●

$$|0\rangle = \frac{1}{\sqrt{2}}|1s\rangle_1|1s\rangle_2\left(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right)$$

antisymmetric w/ exchange ✓

2s, 2p ●

1s ●

Excited State

$$\frac{1}{\sqrt{2}}\left(|1s\rangle_1|2s\rangle_2 - |1s\rangle_2|2s\rangle_1\right)|+\rangle_1|+\rangle_2$$

one of several options

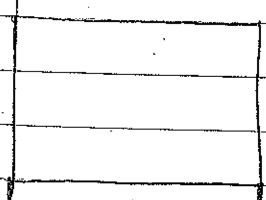
lifetime $\sim 10^4$ s!

requires spin-flip transition.

FOR A FERMION (EXCLUSION PRINCIPLE)
↳ $\Psi(\ldots i \ldots i) = -\Psi(\ldots i \ldots i)$
must be ZERO

▷ What happens with a lot of fermions?

* Box of N identical fermions, free, nonrelativistic *



L

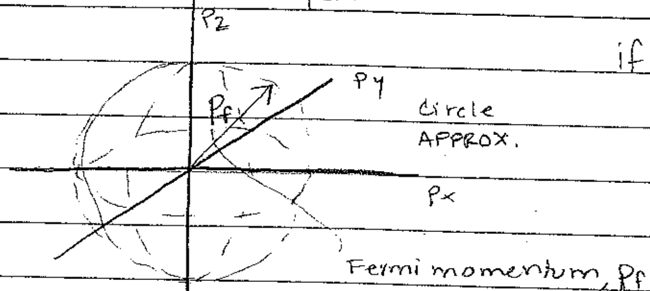
enforce periodicity $e^{ipx/\hbar} = e^{\frac{ip}{\hbar}(x+L)}$

$$\Rightarrow \frac{pL}{\hbar} = 2\pi n \quad n \text{ integer}$$

$$p_x = \frac{2\pi\hbar}{L} n$$

*This implies two orthogonal states (spin-1/2) per volume $\left(\frac{2\pi\hbar}{L}\right)^3$ in momentum space.

$$E = \sum_{\substack{\text{all} \\ \text{particles}}} \frac{p_i^2}{2m} \Rightarrow \text{To minimize } E, \text{ need to minimize } \bar{p}_i^2$$



if $\frac{2\pi\hbar}{L} \ll |P_{max}|$, then a spherical smooth approx. is accurate

Relation between N & p_F?

$$N = \frac{\overbrace{2}^{\text{2 electrons}} \cdot \overbrace{\frac{4}{3}\pi p_F^3}^{\text{total vol. of sphere}}}{\underbrace{\left(\frac{2\pi\hbar}{L}\right)^3}_{\text{vol. of 1 state}}}$$

$$\hookrightarrow N = \frac{1}{3\pi^2}\left(\frac{p_F L}{\hbar}\right)^3$$

This Implies that

$$\underbrace{n}_{\#\text{ density}} = \frac{N}{V} = \frac{1}{3\pi^2}\frac{p_F^3}{\hbar^3}$$

$\rightarrow$ What about energy (density)? $\frac{E}{V} = \frac{3^{5/3}\pi^{4/3}}{10}\frac{\hbar^2}{m} n^{5/3}$

(?) FERMI PRESSURE: $\frac{dE}{dV} < 0$
(also degeneracy pressure)

Relativistic energy: $E = \sqrt{p^2c^2 + m^2c^4}$ } Pressure in relativ. is less that non rel
$\rightarrow E \sim pc$ when $p \gg mc$

Chandrasekhar limit: for $M > 1.4\,M_\odot$ relativistic energy/mom. corrections.
$\Rightarrow$ star collapses.