Lecture 2 - Modernizing Young's Experiment 09/02/15

1. Photon Interferometers
2. Complex Amplitudes

⇨ From Lecture 1:

At a microscopic level, you have to give up on classical probability P and replace it with a probability amplitude φ

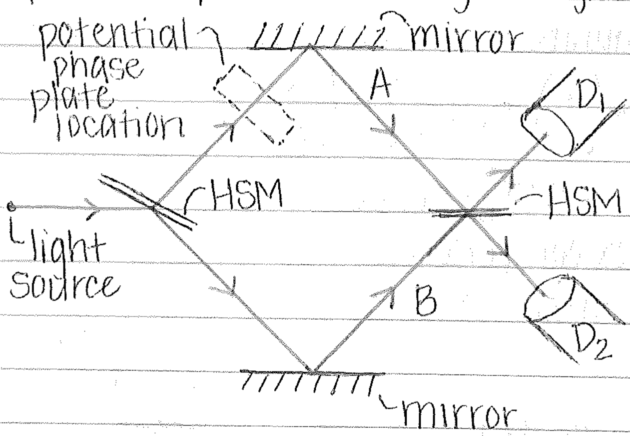
- $P = |\varphi|^2$
- Obeys superposition principle; $\varphi_{tot}(\vec{r}) = \varphi_A(\vec{r}) + \varphi_B(\vec{r})$
- No passive observer (measurement is key)

} (simplified) axioms of Quantum Mechanics

Must modernize Young's Double slit experiment → interferometers

1. Photon Interferometers

"packets of photons" traveling through system



HSM: half silvered mirror - beam splitter; can reflect or transmit with some probability

Components - all except detectors exist in classical mechanics

① Detector (perfect single photon detector)

→ $P_a \propto$ rate of clicks (detections)

$\propto |\varphi_a|^2$

② Phase Plate (not pictured above)

Classically:

$E \to$ [plate] $\to Ee^{i\delta}$

$e^{i\delta}$ = phase shift

$\vec{E}(x,t) = \hat{x}\sin(kx - \omega t) \to \hat{x}\sin(kx - \omega t - \delta)$

Quantum Mechanically:

$\varphi_{in} \to$ [plate, width W] $\to \varphi_{out}$

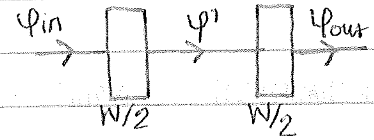
$|\varphi_{in}|^2 = |\varphi_{out}|^2$ Probability photon in = Probability photon out

⇒ Normalization = 1, 100% probability

$|\varphi_{in}|^2 = |\varphi_{out}|^2 \Rightarrow \varphi_{out} \propto \varphi_{in}$ (up to a sign, or "phase")

- W causes $\varphi_{out} = -\varphi_{in}$

What does $W/2$ do?



$\varphi_{out} = -\varphi_{in}$

$\Rightarrow \varphi' = i\varphi_{in}$

└ most phases are complex in nature

We cannot express φ like E because it <u>must be normalizable</u> for all of spacetime (and it must normalize to 1)

2. Complex Amplitudes

<u>Representing Two Paths</u>

- the split of paths is key to the system

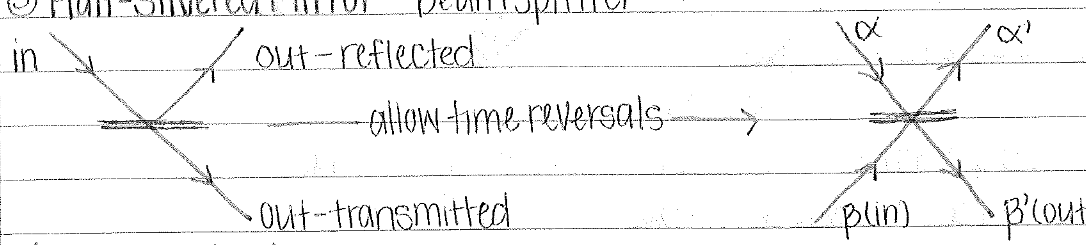
$\underbrace{\alpha}(\text{sit. A}) + \underbrace{\beta}(\text{sit. B})$ — probability amplitudes that it will be in A or B

$$= \alpha\begin{pmatrix} \rightarrow \\ \emptyset \end{pmatrix} + \beta\begin{pmatrix} \emptyset \\ \rightarrow \end{pmatrix} = \alpha\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

└ Complex vector space ∩ Hilbert space

→ multiple complex components

③ Half-Silvered Mirror - beam splitter



$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \xrightarrow{B.S.} \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix}$ — beam splitter takes a vector in and produces a vector out

By the superposition principle:

$$\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} \xrightarrow{B.S.} \begin{pmatrix} \alpha_1' \\ \beta_1' \end{pmatrix} + \begin{pmatrix} \alpha_2' \\ \beta_2' \end{pmatrix}$$

$\Rightarrow \alpha' = w\alpha + y\beta$ — α' & β' have to be linear in α and β

$\beta' = \underbrace{x\alpha + z\beta}_{\text{weights}}$ — α' depends on α and β because part of β may be transmitted, where y is the transmission amplitude (probability) for β

$$\Rightarrow \begin{pmatrix} \alpha' \\ \beta' \end{pmatrix} = \underbrace{\begin{pmatrix} w & y \\ x & z \end{pmatrix}}_{\text{Beam splitter matrix}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where w & z are reflection amplitudes and y & x are transmission amplitudes

All components (excepting detectors) can be represented as matrices!

Phase plate: photon through A phase shifted

$$\boxed{} = \begin{pmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{pmatrix}$$

photon through B unchanged

Mirror:

$$/////// = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \leftarrow \pi \text{ phase shift}$$

- shift not required; can imagine mirror & phase plate to represent as all positive