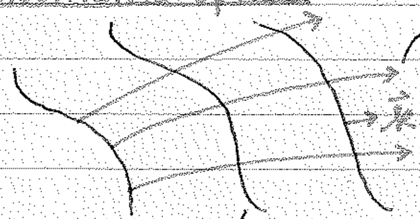
Lecture 19 - Reflections & Interfaces 04/07/16

1. Geometric Optics
2. Reflections @ Interfaces
3. Guided Waves

1. Geometric Optics



paths along which energy travels
(normal to phase fronts)
$\vec{k}$ and $\vec{v}_g$ need not be parallel

constant phase lines

Evolution of Trajectory: $\omega(\vec{x}, \vec{k})$

trajectory follows group velocity

$$\frac{d\vec{x}}{dt} = \frac{\partial \omega}{\partial \vec{k}}\Big|_{\vec{x}} = \vec{v}_g$$

The wave vector, $\vec{k}$, will evolve along this trajectory

$$\frac{d\vec{k}}{dt} = \frac{-\partial \omega(\vec{x}, \vec{k})}{\partial \vec{x}}\Big|_{\vec{k}}$$

*note: this is true for any kind of wave

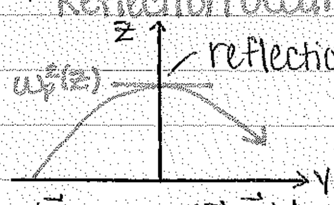
For electromagnetic waves

$k^2 = \omega^2 \varepsilon \mu \quad \mu = \mu_0, \ \varepsilon = \varepsilon_0(1 - \omega_p^2/\omega^2)$

$$\hookrightarrow k^2 = \frac{\omega^2}{c^2}\left(1 - \underbrace{\frac{\omega_p^2}{\omega^2}}\right)$$

$$\omega = \sqrt{k^2 c^2 + \omega_p^2(z)}$$

→ Reflection occurs for $\omega = \omega_p(z)$

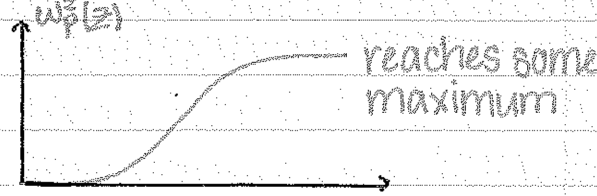


$$\frac{d\vec{k}}{dt} = \frac{-\partial \omega(\vec{x}, \vec{k})}{\partial \vec{x}}\Big|_{\vec{k}}$$

$$\frac{d}{dt} k_y = -\frac{\partial \omega}{\partial y} = 0$$

y is an "ignorable coordinate"

$$\frac{dk_z}{dt} = -\frac{\partial \omega}{\partial z} = \frac{-1}{2\omega}\frac{\partial}{\partial z}\omega_p^2(z)$$



$$\frac{d\vec{x}}{dt} = \frac{\partial \omega}{\partial \vec{k}} = \frac{1}{2\omega} 2\vec{k}c^2$$

$$= \frac{kc^2}{\omega}$$ } Here, the wave vector $\vec{k}$ is parallel to $\vec{v}_g$
→ dependence in the local dispersion relation only on k^2 and not let $k_{x,y,z}$ indicates an isotropic medium in all directions

$$\frac{dz}{dt} = \frac{k_z c^2}{\omega} \; ; \quad \frac{dy}{dt} = \frac{k_y c^2}{\omega}$$

↓ solving these equations ↓

$$k_z(z) = \pm\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_p^2(z)}{c^2} - k_y^2}$$

this yields a turning point ω_p^2

$$\omega_p^2(z_t) = \omega^2 - \underbrace{k_y c^2}$$

the larger this propagation constant in y, the lower in z-altitude the reflection will occur (where the highest reflection point is for directly vertical propagation)

full determination:

$$\frac{d}{dt}\omega = \frac{\partial \omega}{\partial t} + \underbrace{\frac{d\vec{x}}{dt}\cdot\frac{\partial \omega}{\partial \vec{x}} + \frac{d\vec{k}}{dt}\cdot\frac{\partial \omega}{\partial \vec{k}}}_{=0 \text{ for constant medium}}$$

What if $\varepsilon > \varepsilon_0$?

$\mu = \mu_0$, homogeneous medium

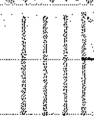
$\omega^2 n^2 = k^2 c^2$

↳ n = index of refraction $= \frac{c}{v_\phi} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$

v_ϕ: phase velocity

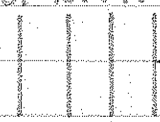
→

for high n:



→ phase fronts closer together

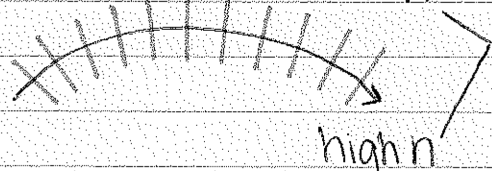
for low n:



→ phase fronts farther apart

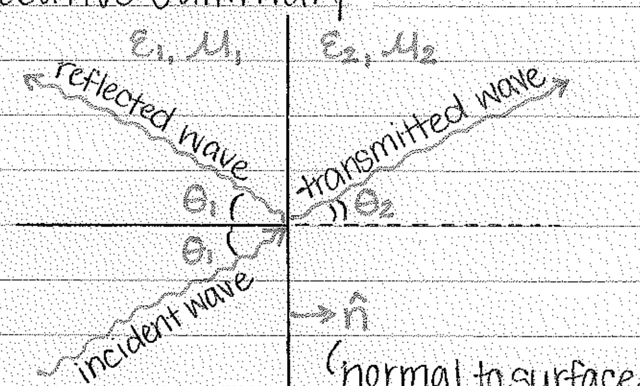
For non-homogeneous materials:

$$\frac{d\vec{k}}{dt} = \left.\frac{-\partial \omega}{\partial \vec{x}}\right|_{\vec{k}_0} = \frac{\omega}{n}\nabla n \Rightarrow$$ $\vec{k}$ will bend towards regions of high n values



2. Reflections at Interfaces

·The executive summary·



Variables continuous across the boundary:

$\vec{E}_{\perp_1} = \vec{E}_{\perp_2}$

$\perp$ to normal $\hat{n}$ = ∥ to interface surface

$\vec{H}_{\perp_1} = \vec{H}_{\perp_2}$

Where the ratio of E:H for a plane wave in the medium gives the impedance of the medium.

$$\left(\frac{E_{t_1}}{H_{t_1}}\right) = Z_1 \quad , \quad \left(\frac{E_{t_2}}{H_{t_2}}\right) = Z_2$$

transverse component

$$\Rightarrow \rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$ The electric field reflection coefficient (ratio of reflected $\vec{E}$ to incident $\vec{E}$)

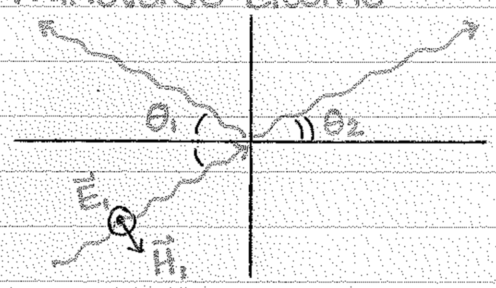
Imposing these continuities automatically satisfies
$\hat{n}\cdot\nabla\times\vec{H} = -i\omega\vec{D}\cdot\hat{n}$

→ So for normal incidence
* polarization does not matter here as $\hat{n}\,\|$ direction of propagation

$$Z_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}, \quad Z_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

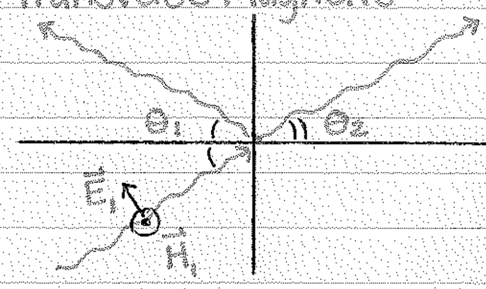
What about for oblique incidence?
→ Polarization now affects which components of the field are tangential when you reach the surface

· Transverse Electric ·



$$Z_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}\frac{1}{\cos(\theta_1)}$$

· Transverse Magnetic ·



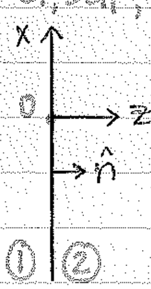
$$Z_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos(\theta_1)$$

the ratio of the components that will be tangential at the surface, not the full wave as with normal incidence

ex. Normal Incidence

frequency will be the same in both regions

$\varepsilon_1, \mu_1, \; k_1 = \omega\sqrt{\varepsilon_1\mu_1}, \; \eta_1 = \sqrt{\mu_1/\varepsilon_1}$



incident / reflected

① $E_x = \mathrm{Re}\{\hat{E}_{1i}\exp(ikz - i\omega t) + \hat{E}_{1r}\exp(-ikz - i\omega t)\}$

reflected H changes sign

$H_y = \mathrm{Re}\left\{\frac{\hat{E}_{1i}}{\eta_1}\exp(ik_1z - i\omega t) - \frac{\hat{E}_{1r}}{\eta_1}\exp(-ik_1z - i\omega t)\right\}$

"most important minus sign of the semester"

② $E_x = \mathrm{Re}\{\hat{E}_{2t}\exp(ik_2z - i\omega t)\}$

$H_y = \mathrm{Re}\left\{\frac{\hat{E}_{2t}}{\eta_2}\exp(ik_2z - i\omega t)\right\}$

We now want to equate fields transverse to the normal @ $z=0$, the boundary

$$\hat{E}_{1i} + \hat{E}_{1r} = \hat{E}_{2t}$$

$$\frac{1}{\eta_1}(\hat{E}_{1i} - \hat{E}_{1r}) = \frac{1}{\eta_2}\hat{E}_{2t} = \frac{1}{\eta_2}\underbrace{(\hat{E}_{1i} + \hat{E}_{1r})}$$

$$\frac{\hat{E}_{1r}}{\hat{E}_{1i}} = \rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad -1 \le \rho \le 1$$

- $\rho \to -1$ for $\eta_2 \to 0$, which occurs for ε_2 very large (large ε like a conductor)

$\rho < 0$: traveling from low ε_1 to high ε_2

$\rho > 0$: traveling from high ε_1 to low ε_2

$\rho = 0$: $\varepsilon_1 = \varepsilon_2$

What about power transmission?

$$\vec{S} = \vec{E} \times \vec{H}$$

incident power

Power reflection condition:

$$\langle S_z \rangle_1 = \frac{1}{2}\left\{\frac{|\hat{E}_{1i}|^2}{\eta_1} - \frac{|\hat{E}_{1r}|^2}{\eta_1}\right\}$$

reflected power

$$R = |\rho|^2 \to |\rho|^2 |\hat{E}_{1i}|^2 / \eta_1$$

$\langle S_z \rangle = 0$ here

For electric field transmission

$$\hat{E}_{2t} = (1+\rho)\hat{E}_{1i}$$

no reflection for $\rho = 0$

$$\langle S_z \rangle_2 = \frac{1}{2}\frac{|\hat{E}_{2t}|^2}{\eta_2}$$

define: Transmission coefficient, τ

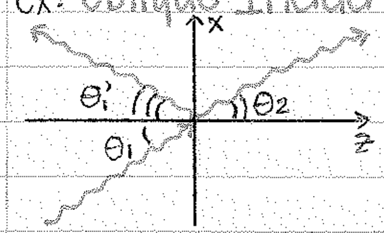
$$\tau = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

ratio of transmitted to incident power

$$\frac{S_t}{S_i} = \frac{|\hat{E}_{2t}|^2}{\eta_2}\frac{\eta_1}{|\hat{E}_{1i}|^2} = \frac{\eta_1}{\eta_2}\left(\frac{2\eta_2}{\eta_1 + \eta_2}\right)^2$$

$$= 1 - |\rho|^2 = 1 - R$$

ex: Oblique Incidence



$$\vec{E}_1 = \mathrm{Re}\{\hat{E}_i \exp[ik_1(\cos(\theta_1)z + \sin(\theta_1)x) - i\omega t]$$
$$+ \hat{E}_r \exp[-ik_1(\cos(\theta_i')z) + ik_1(\sin(\theta_i')x) - i\omega t]\}$$

$$\vec{E}_2 = \mathrm{Re}\{\hat{E}_t \exp[ik_2(\cos(\theta_2)z) + ik_2(\sin(\theta_2)x) - i\omega t]\}$$

*recall: these relations only hold for constant ω

By SNELL's LAW:

$k_1 \sin(\theta_1) = k_2 \sin(\theta_2)$

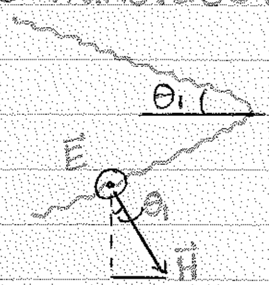
→ must also hold for reflection $= k_1 \sin(\theta_i')$

$\Rightarrow k_1 \sin(\theta_1) = k_1 \sin(\theta_i')$, $\theta_i = \theta_i'$

Types of Polarization:

- let the page be the plane of incidence

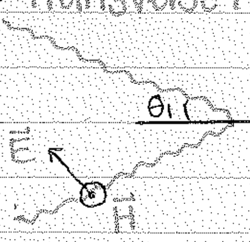
① Transverse Electric



Electric Field out of the plane of incidence

$$Z_1 = \frac{|E_{\perp}|}{|H_{\perp}|} = \frac{\eta_1}{\cos(\theta_1)} \longrightarrow Z_2 = \frac{|E_{\perp 2}|}{|H_{\perp 2}|} = \frac{\eta_2}{\cos(\theta_2)}$$

② Transverse Magnetic



Magnetic Field out of the plane of incidence

$$Z_1 = \eta_1 \cos(\theta_1) \longrightarrow Z_2 = \eta_2 \cos(\theta_2)$$

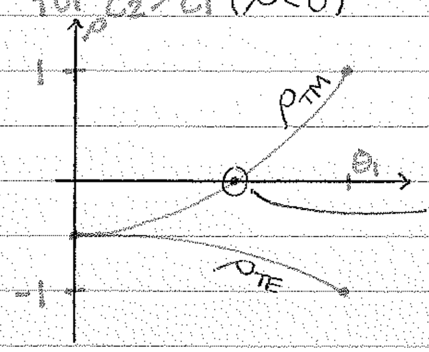
Where the reflection coefficient still holds

$$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

then, assuming $\mu_1 = \mu_2$

→

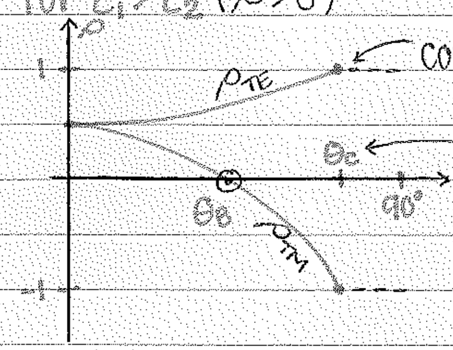
• for $\varepsilon_2 > \varepsilon_1$ ($\rho < 0$)



Brewster's Angle: the angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface, with no reflection

$$\tan(\theta_B) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

• for $\varepsilon_1 > \varepsilon_2$ ($\rho > 0$)



complex past this point

Critical Angle: the condition for total internal reflection

$$\sin^2(\theta_c) = \frac{\varepsilon_2}{\varepsilon_1}$$

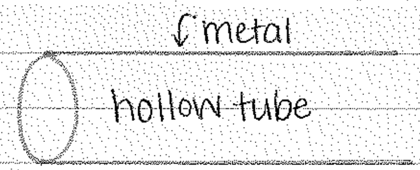
note that it is not possible to have a θ_c for $\varepsilon_2 > \varepsilon_1$

--------END OF MIDTERM II MATERIAL----------

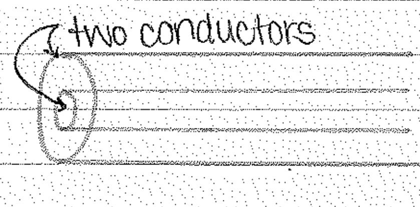
3. Guided Waves

* Frequency, mode of propagation, material impedance, and loss constants will all impact effectiveness of transmission

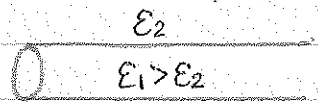
Method #1: Metallic Waveguides



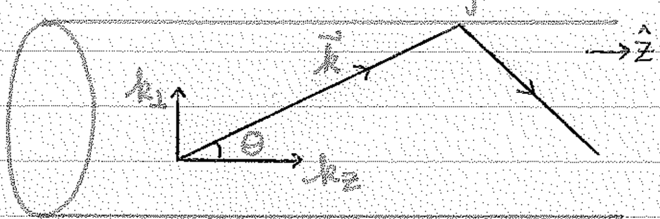
Method #2: Transmission Line



Method #3: Optical Fiber



Method #1: Metallic Waveguides



$k_\perp$ - transverse component

k_z - axial component

back to basics

$\lambda = c/f$, $\lambda = 2\pi/k$

$\omega = 2\pi f \rightarrow \omega = kc$

$$\omega^2 = \underbrace{k_\perp^2}_{k_\perp^2 \text{ is quantized}} c^2 + k_z^2 c^2$$