Lecture 1 – Logistics & Lagrangians 09/01/15

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Course Outline

I. Formalism
- Lagrangian Approach (ch 1 & 2)
- Hamiltonian Approach (ch 8)

II. Basic Applications (ch 1.5, 3, 3.10, 3.11, 4, 5 & 6)
III. Special Theory of Relativity (ch 7)
IV. TBD Additional Material (ch 11, 12 & 13)

Motivation for this course:

- not really a "research field"
- concepts from PHYS601 useful indirectly
  e.g. Quantum Mechanics – start with classical mechanics/ Hamiltonian formulation → impose commutation relations on variables to transition into Q.M.
- useful in other fields
  e.g., thermodynamics, statistical physics
- practical – qualifier exam

I. Formalism: Lagrangian Approach

Why not just use Newton's laws ($F=ma$)?
"inelegant"; no quantum crossover

→ Practically speaking: Newton's laws are valid only in inertial reference frames and are cumbersome when dealing with extended objects (as opposed to point particles)

→ Newton's laws "hide" structure/symmetries of classical mechanics – which is why chaos theory was not developed for 200 years post-Newton

→

Lagrangian Formulation

$\mathcal{L}$ for a single free particle $= \underbrace{\tfrac{1}{2}mv^2}_{\text{just kinetic energy, } T}$; $v = |\dot{\vec{r}}|$ where $\dot{} = \frac{d}{dt}$

$\mathcal{L}$ for a single interacting particle $= T \underbrace{- V(\vec{r})}_{\text{subtract the interaction potential}}$

Generalize to $q_i(t)$:

$\mathcal{L}(q_i(t); \dot{q}_i(t))$

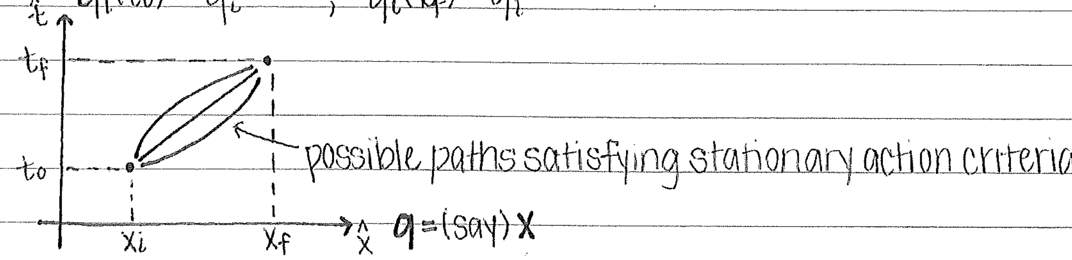
⇒ **Must be able to "recover" Newton's law via Lagrange's equations** (Euler's equations) **in turn from principle of stationary action/ Hamilton's principle**

a.k.a. principle of least action

For stationary action:

- smooth paths with fixed endpoints

$q_i(t_0) = q_i^{initial}$, $q_i(t_f) = q_i^{final}$



Action (I or S):

$I = \int_{t_0}^{t_f} \mathcal{L}(q, \dot{q})\, dt$

assigns a number to each path (function q as a function of time $q(t)$)

⇒ functional

→ Actual path is the extremum of I.

consequence of the principle of least action

Variational Principle

$q_i(t) \rightarrow q_i(t) + \underbrace{\delta q_i(t)}_{\text{function -- small variation in path}}$; $\delta q_i(t_0) = \delta q_i(t_f) = 0$

$$\delta I = \int_{t_0}^{t_f} \delta\mathcal{L}\, dt = \int_{t_0}^{t_f} \left[\left(\frac{\partial \mathcal{L}}{\partial q_i}\right) \delta q_i + \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) \delta \dot{q}_i \right] dt$$

Change in the action under this small variation in path

$$\delta I = \int_{t_0}^{t_f} dt \left[\left(\frac{\partial \mathcal{L}}{\partial q_i}\right)\delta q_i + \left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) \underbrace{\frac{d}{dt}}(\delta \dot{q}_i) \right]$$

small shift/variation in $\dot{q}$

move derivative to other function (adds −)

$$0 = \int dt \left[\left(\frac{\partial \mathcal{L}}{\partial q_i}\right)\delta q_i - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)\delta \dot{q}_i \right] + \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)\delta q_i \Big|_{t_0}^{t_f}}_{\text{boundary term}}$$

Stated: $\delta q = 0$ @ endpoints

extremum: $I \to 0$

(first derivative to 0)

$$0 = \int dt \underbrace{\left[\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right)\right]}_{=0} \delta q_i$$

variation in q is arbitrary (except at endpoints)