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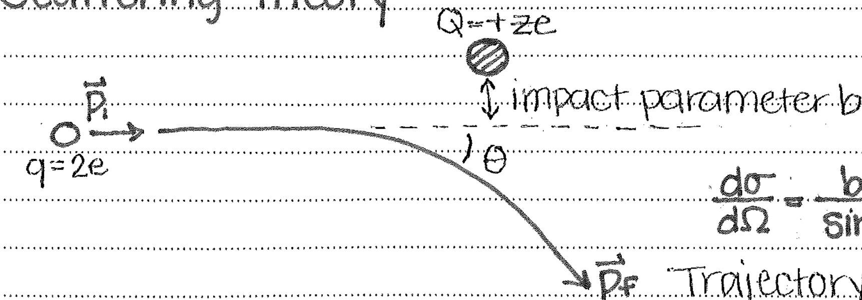
Rutherford Scattering

Scattering Theory



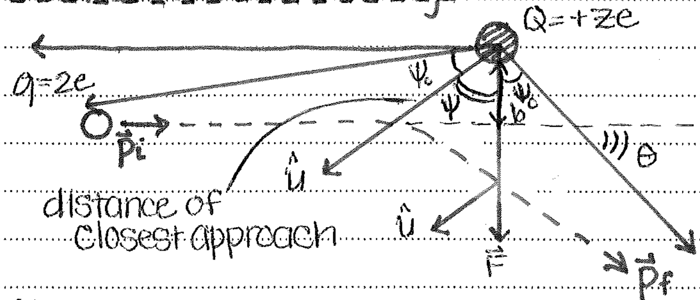
$$\frac{d\sigma}{d\Omega} = \frac{b(\theta)}{\sin(\theta)}\left|\frac{db}{d\theta}\right|$$

Trajectory: $b(\theta) \Rightarrow d\sigma/d\Omega$

Scattering Cross-Section

$$\sigma = \iint \frac{d\sigma}{d\Omega}\, d\Omega$$

Rutherford Scattering



Hyperbola Trajectory

→ Calculate $b(\theta)$

$$F = \frac{Qq/4\pi\varepsilon_0}{r^2} \equiv \frac{\gamma}{r^2}$$

initial: $\vec{p}_i$, $\psi = -\psi_0$

final: $\vec{p}_f$, $\psi = +\psi_0$

$2\psi_0 + \theta = \pi$

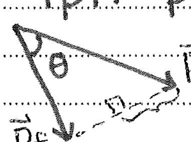
→ 2 ways to calculate the $\Delta\vec{p}$ of the α-particle

$\Delta p = \vec{p}_f - \vec{p}_i$

1) Energy Conservation

$$\frac{\vec{p}_i^{\,2}}{2m} = \frac{\vec{p}_f^{\,2}}{2m}$$

$\Rightarrow |\vec{p}_i| = |\vec{p}_f| = p$



$$\frac{|\Delta\vec{p}|}{2} = p\sin(\theta/2)$$

$|\Delta\vec{p}| = 2p\sin(\theta/2)$

2) Impulse - Momentum Theorem

$\Delta\vec{p} = \int \vec{F}\,dt$

$\Delta\vec{p}\cdot\hat{u} = |\Delta\vec{p}| = \int \vec{F}\cdot\hat{u}\,dt = \int F\cos(\psi)\,dt = \int \gamma/r^2 \cos(\psi)\,dt$

$dt \rightarrow dt/d\psi\; d\psi = 1/\dot{\psi}\, d\psi$

Conservation of Angular Momentum

$$\ell = bp = mr^2\dot{\psi}$$

$$\dot{\psi} = \frac{bp}{mr^2}$$

$$|\Delta\vec{p}| = \int_{-\psi_0}^{\psi_0} \frac{\gamma}{\cancel{r^2}} \cos(\psi) \frac{m\cancel{r^2}}{bp} d\psi$$

$$= \frac{\gamma m}{bp} \int_{-\psi_0}^{\psi_0} \cos(\psi)\, d\psi$$

$$|\Delta\vec{p}| = \frac{2\gamma m}{bp} \sin(\psi_0)$$

Compare $|\Delta\vec{p}|$s to get b

$$|\Delta\vec{p}| = |\Delta\vec{p}|$$

$$2p\sin(\theta/2) = \frac{2\gamma m}{bp} \underbrace{\sin(\psi_0)}_{\sin(\pi/2 - \theta/2) = \cos(\theta/2)}$$

$$b(\theta) = \frac{\gamma m}{p^2} \frac{\cos(\theta/2)}{\sin(\theta/2)}$$

$$\frac{d\sigma}{d\Omega} = \frac{b(\theta)}{\sin(\theta)} \left|\frac{db}{d\theta}\right|$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\gamma m}{p^2}\right)^2 \frac{\cos(\theta/2)}{\sin(\theta/2)2\sin(\theta/2)\cos(\theta/2)} \left|\left(\frac{-1}{2}\right)\csc^2(\theta/2)\right|$$

$E = p^2/2m$

$$= \left(\frac{\gamma}{4E\sin^2(\theta/2)}\right)^2$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{Qq/4\pi\varepsilon_0}{4E\sin^2(\theta/2)}\right)^2$$

$F = \gamma/r^2$

$$\sigma = \int_0^{2\pi}\int_0^{\pi} \frac{d\sigma}{d\Omega} \sin(\theta)\, d\theta\, d\Omega = \infty$$