Zombie Apocalypse: An Epidemic Model

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Epidemic Models

Epidemic models offer significant insight into predicting and controlling infectious diseases, such as measles and influenza (1-6). Epidemic models describe disease dynamics via susceptible, infected, and recovered (SIR) subgroups with parameters related to rate of infectiousness (β) and recovery (ζ); these can be modified to different infectious disease dynamics.

$$S' = -\beta SI$$

$$I' = \beta SI - \zeta R$$

$$R' = \zeta R$$

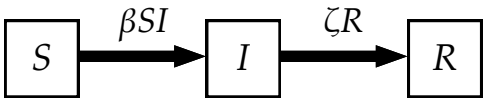


Fig. 1. Basic epidemic dynamics

Similar to predator-prey dynamics, SIR models deal with interacting terms in which one subgroup benefits from interaction and the other is hindered. However, unlike predator-prey models, S+I+R=N, where N is a constant population, resulting in dynamics similar to that seen in Figure 2.

Goals of this paper

In this study, we address the shortcomings of previous literature (7) and explore an improved model which is more consistent with dynamics exhibited in movies, such as "Night of the Living Dead" and "Shaun of the Dead". We replicate the simulations of the previous paper, but not with the same parameters in the paper. Our mathematical analysis compares movies to determine if a consistent apocalypse is portrayed or if different popular renditions demonstrate qualitatively different infection dynamics. Finally, we use estimates of the populations in the movies to fit the parameters of the models, and explore the uncertainties in the model predictions.

1. Grais, RF; Ferrari, MJ; Dubray, C; Bjornstad, ON; Grenfell, BT; Djibo, A; Fermon, F; Guerin, PJ. (2006) Estimating transmission intensity for a measles epidemic in Niamey, Niger: lessons for intervention: Transactions of the Royal Society of Tropical Medicine and Hygiene. 100: 867-873.

2. Hooten, MB; Anderson, J; Waller, LA. (2010) Assessing North American influenza dynamics with a statistical SIRS model: Spatial and Spatio-temporal Epidemiology. 1: 177-185.

3. Kuniya, T. (2011) Global stability analysis with a discretization approach for an age-structured multigroup SIR epidemic model: Nonlinear Analysis: Real World Applications. 12: 2640-2655.

4. Li, XZ; Li, WS; Ghosh, M. (2009) Stability and bifurication of an SIR epidemic model with nonlinear incidence and treatment: Applied Mathematics and Computation. 210: 141-150.

5. McGilchrist, CA; McDonnell, LF; Jorm, LR; Patel, MS. (1996) Loglinear models using capture - Recapture methods to estimate the size of a measles epidemic: Journal of Clinical Epidemiology. 49: 293-296.

6. Tuckwell, HC; Williams, RJ. (2007) Some properties of a simple stochastic epidemic model of SIR type: Mathematical Biosciences. 208: 76-97.

7.Munz, P; Hudea, I; Imad, J; Smith, RJ. (2009) When Zombies Attack!: Mathematical Modelling of an Outbreak of Zombie Infection: Infectious Disease Modelling Research Progress. Nova Science Publishers, Inc., pp. 133-150.

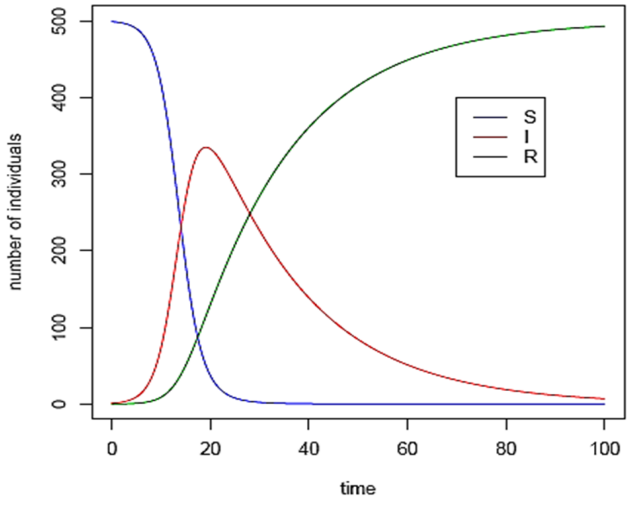


Fig. 2. Typical SIR (susceptible, infected, recovered) epidemic model dynamics (Bonhoeffer, **SIR models of epidemics).**

Replicating the Literature

In "When Zombies Attack!: Mathematical Modeling of an Outbreak of Zombie Infection" (7), the authors apply the standard SIR model to a zombie apocalypse (Fig. 3). However, there are some issues with the zombie example.

1. Models don't match films

In the paper, they model only the "classical pop-culture zombie: slow moving, cannibalistic, and undead." However, according to the films (ex. "Night of the Living Dead," "Dawn of the Dead," and "Shaun of the Dead"), zombies can permanently die by brain damage, being burned alive, or starving. Without a permanent removal term, the zombie population and dead population will constantly cycle (8, Fig.3).

2. All results depend on poor model assumptions

3. No data is used

4. Wrong parameters are given

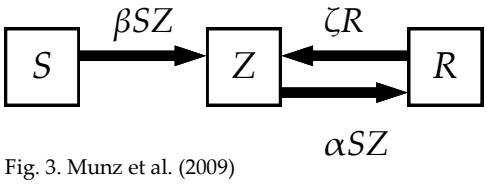


Fig. 3. Munz et al. (2009)
Basic zombie dynamics

8. Slightly modified from traditional SIR model, Munz et al. begins with this simple SZR model in which "Z" stands for "zombie" and the ζR term moves from "removed" (not "recovered") to zombie, hence the term 'living dead.'

$-\alpha SZ$ is when a susceptible maims a zombie, moving them to the "removed" population, although this removal is temporary.

This temporarily removed term is inconsistent with all zombie films. According to this model, no matter how the zombie is killed, it can always be recycled into the zombie population via rate of removed to zombies term ζR.

Fig. 4. Terms used in equations and models

KEY	
S	Susceptible
Z	Zombie
R	Removed
I	Infected
Q	Quarantined
δ	Rate of non-zombie-related death
ζ	Rate of removed to zombies
β	Rate of susceptible to zombie
Π	Birth rate
α	Rate of zombie to removed
ϱ	Rate infected become zombies
σ	Rate zombies are quarantined
γ	Rate quarantined killed
ϰ	Rate infected are quarantined
kn	Kill number * Number of attacks

Fig. 5. Equation for zombie dynamics in different conditions

Basic Model
(Munz et al., Figure 3)

$$S' = -\beta SZ$$
$$Z' = \beta SZ + \zeta R - \alpha SZ$$
$$R' = -\zeta R + \alpha SZ$$

Latent Infection
(Munz et al., Figure 4 and 5)

$$S' = \Pi - \beta SZ - \delta S$$
$$I' = \beta SZ - \rho I - \delta I$$
$$Z' = \rho I + \zeta R - \alpha SZ$$
$$R' = \delta S + \delta I - \zeta R + \alpha SZ$$

Quarantine
(Munz et al., Figure 6 and 7)

$$S' = \Pi - \beta SZ - \delta S$$
$$I' = \beta SZ - \rho I - \delta I - \varkappa I$$
$$Z' = \rho I + \zeta R - \alpha SZ - \sigma Z$$
$$R' = \delta S + \delta I - \zeta R + \alpha SZ + \gamma Q$$
$$Q' = \varkappa I + \sigma Z - \gamma Q$$

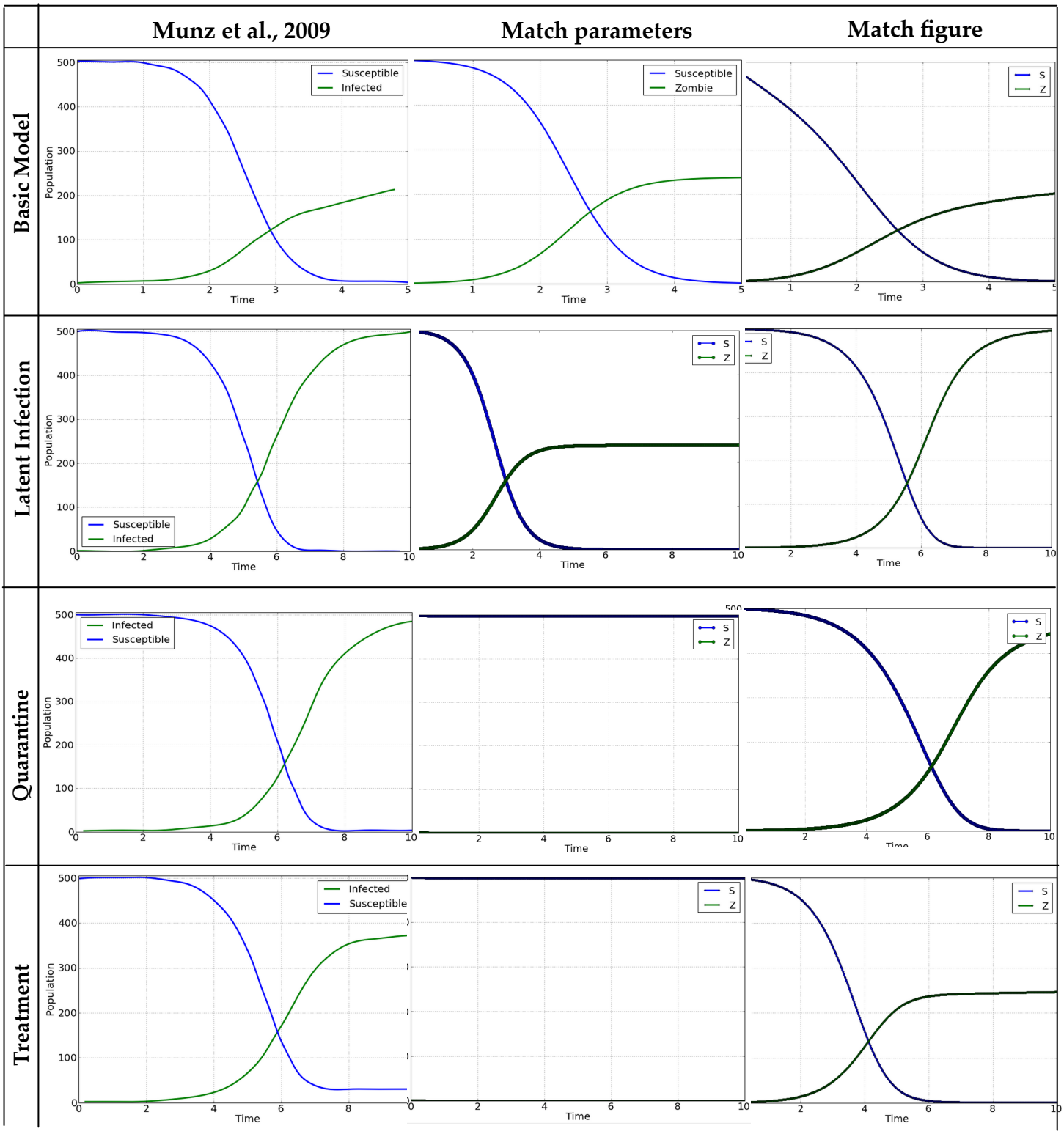
Treatment
(Munz et al., Figure 8 and 9)

$$S' = \Pi - \beta SZ - \delta S + cZ$$
$$I' = \beta SZ - \rho I - \delta I$$
$$Z' = \rho I + \zeta R - \alpha SZ - cZ$$
$$R' = \delta S + \delta I + \alpha SZ - \zeta R$$

Replicating the Literature, continued

In attempting to replicate the figures in the Munz et al. (2009) paper, we discovered that beyond the intrinsically bad mathematical models, the paper also makes programming errors, including parameters off by a factor of 1,000.

Fig. 6. Simulations of different zombie models, as noted by left-most column. Of the three simulations, "Munz" represents the models from the paper, "Match equation" refers to me running the paper's model in Python using the equations and parameters provided in the paper, and "Match model" refers to me running the paper's model in Python and altering parameters in order to look like the paper's models.



Major Findings Zombie infection would likely be disastrous for civilization, although not inevitable as Munz et al. (2009) suggestions. Data are necessary to make reasonable models and parameter estimations

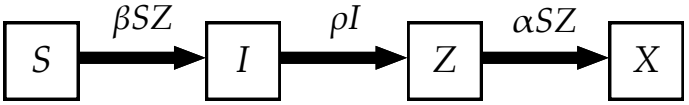


Fig. 7. Proposed model based off "Shaun of the Dead" with susceptible (*S*), infected (*I*), zombie (*Z*), and removed (*X*). Other media that has similar dynamics (with varying parameter estimations) would include Walking Dead, Zombieland, 28 Days Later, and Resident Evil.

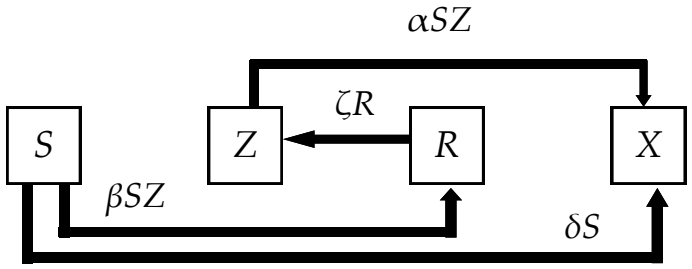


Fig. 8. Proposed model based off "Night of the Living Dead" with susceptible (*S*), zombie (*Z*), temporarily removed (*R*), and permanently removed (*X*).