ENEE 425

Section 0101 Fall 2004

Homework #12

You must attach a printed version of Matlab code (a Matlab file is preferable to copying and pasting a session, if possible). You may put several problems into a single printout, if the start of each problem is made obvious. Don't forget to answer the verbal questions too.

1) Consider $x[n] = \cos(\omega_1 n) + 0.75\cos(\omega_2 n)$,

   a) where $\omega_1 = \frac{2\pi}{14}$ and $\omega_2 = \frac{4\pi}{15}$, and a rectangular window function $w[n]$ which is $N = 64$ samples long. The measured data is $v[n] = w[n]x[n]$. Plot the Fourier transform $V(e^{j\omega})$ for ω in the range $[0,\pi]$. Recall that $V(e^{j\omega})$ is not the same a $V[k]$, so if you wish to plot an approximation of $V(e^{j\omega})$, you should pad $v[n]$ with many zeros before taking its discrete Fourier transform (see Figure 10.9 in the text for a good example).

      i) What are the expected position of the two peaks, and what are the actual positions of the two maxima?

      ii) What is the expected ratio of the two maxima (smaller over larger), and what is the actual ratio?

      iii) What is the expected resolution in ω based on $N = 64$, how does that compare to $\Delta\omega = \omega_2 - \omega_1$?

   b) Repeat (a) but with a 64 point hamming window. Which estimates of the true values improve significantly and which worsen significantly? Which plot looks "cleaner"?

   c) Repeat (a) (with a rectangular window) but with $\omega_2 = \frac{4\pi}{19}$. Which estimates of the true values improve significantly and which worsen significantly?

   d) Repeat (c) but with a 64 point hamming window. Which estimates of the true values improve significantly and which worsen significantly? Which plot looks "cleaner"?

2) Consider problem (1)(d), but with a Kaiser window. With different values of the parameter β, different estimates of the ratio of the peaks can be obtained. Find a value of β which gives a correct estimate of the ratio of the maxima, to 3 significant figures.

3) Replicate Figure 10.9 (pp. 711, 712) in the text. Use `stem()` for subplots a-c and `plot()` for subplot d.

4) Estimating frequency as a function of time is a foreign concept to an ordinary Fourier transform, but not for a time-windowed Fourier transform. Let

$$x[n] = \cos(\omega_1 n + a_1 \sin(\omega_2 n)) \text{ where } \omega_1 = \frac{\pi}{4},\ a_1 = 1000 \text{ and } \omega_2 = \frac{\pi}{8000}$$

for `n = [0:16000-1];`. This is a sinusoidally frequency modulated (SFM) signal.

a) Plot the `specgram(x)`, which is a moving windowed Fourier transform in the signal processing toolbox. Interpret the figure in words. What are the units of the x axis? the y axis? You can fix the axes by changing the third argument to specgram: `specgram(x,[],1);` (the [] in the second argument means use the default value).

b) You can change the window size by changing the second value, called `NFFT`—the default is 256. What happens to the horizontal resolution when you change the 2nd argument to 128 or 512? The vertical resolution?

c) You can change the window from Hanning (the default) to anything else you want, by giving the window as a vector that is `NFFT` points long. Using `NFFT = 256` (the default), what changes when you give if a boxcar (rectangular) or hamming window? Describe especially the width of the band and the “streakiness” of the band (which correspond to the width of the main lobe and the amplitude of the side lobes of the particular window you use).

[The fourth argument of specgram is the number of points from each window that overlap—the default is `NFFT /2` meaning that half of each window is used in the next window. You may wish to see what effect this has on your own.]

d) Repeat (a) but first add white noise to the signal: `+ 0.5*randn(size(n));`. Again with more noise, `+ 3*randn(size(n));`. How much noise can you add before the signal is lost? The time windowed Fourier transform is often used to estimate properties of noisy signals.