Homework #8

1) Consider a causal impulse response $h[n]$ defined implicitly by $y[n]+a_1y[n-1]=b_0x[n]+b_1x[n-1]$, where $a_1=2, b_0=1, b_1=-5$.

a) Evaluate the transfer function $H(z)$ and its ROC.

b) Justify the statement that $H(z)$ is not stable and does not have a stable inverse.

c) Find a causal transfer function $H_{\min}(z)$ and its ROC such that $\left|H_{\min}(e^{j\omega})\right|=\left|H(e^{j\omega})\right|$ but with $H_{\min}(z)$ minimum phase (i.e. stable with a stable inverse). Be careful to find the correct overall gain factor.

d) Consider the causal impulse response $h_{\min}[n]$ which is the inverse Z transform of $H_{\min}(z)$. Without calculating $h_{\min}[n]$ explicitly, find the constants $\tilde{a}_1,\tilde{b}_0,\tilde{b}_1$ so that $h_{\min}[n]$ is defined implicitly by $y[n]+\tilde{a}_1y[n-1]=\tilde{b}_0x[n]+\tilde{b}_1x[n-1]$

e) One reason to compute $H_{\min}(z)$ is to undo the distortion gain of $H(z)$. For this we need $H^I_{\min}(z)$, the inverse of $H_{\min}(z)$. Evaluate $H^I_{\min}(z)$. For $H^I_{\min}(z)$ causal, is it stable?

f) What is the magnitude of the transfer function of the **total** system defined by applying $H(e^{j\omega})$ and then following it with $H^I_{\min}(e^{j\omega})$? Simplify.

2) For each of the following causal, allpass filters, answer the subsequent questions. Feel free to use Matlab.

a) $H(z)=\dfrac{z^{-1}-a^*}{1-az^{-1}}$, where $a=1/8$

b) $H(z)=\dfrac{z^{-1}-a^*}{1-az^{-1}}$, where $a=-1/8$

c) $H(z)=\dfrac{z^{-1}-a^*}{1-az^{-1}}\dfrac{z^{-1}-a}{1-a^*z^{-1}}$, where $a=j/8$

i) Plot the gain $\left|H(e^{j\omega})\right|$ in dB for ω in the range $[0,2\pi]$.

ii) Plot the phase in degrees for ω in the range $[0,2\pi]$.

iii) Plot the unwrapped phase in degrees for ω in the range $[0,2\pi]$. Is this consistent with the answers in (ii)? Why?

iv) Plot the group delay (in samples) for ω in the range $[0,2\pi]$. Is it always positive?

3) Consider the FIR system described by $H(z) = (1 - az^{-1})(1 - a^{*}z^{-1})$ for $a = -4j$.

a) Is it causal? Is it stable? Is its causal inverse stable? Explain.

b) Plot the gain $|H(e^{j\omega})|$ in dB for ω in the range $[0, 2\pi]$.

c) Find a compensating filter $H_c(z)$ such that $|H(e^{j\omega})H_c(e^{j\omega})| = 1$ and $H_c(z)$ is stable. Hint: Decompose H into minimum phase and allpass components, and invert the minimum phase component.