

**Homework #6**

1) For each of the following functions in  $z$  space, compute the inverse Z transform  $x[n]$ .

a)  $X(z) = 3$

b)  $X(z) = \frac{z}{3}$

c)  $X(z) = 5z^{-2}$

d)  $X(z) = \frac{1}{1 - z^{-1}/4}$  for  $|z| > 1/4$

e)  $X(z) = \frac{4}{3 + z^{-1}}$  for  $|z| > 1/3$

f)  $X(z) = \frac{z}{1 - 2z^{-1}}$  for  $|z| > 2$

2) For each of the following functions in  $z$  space, compute the inverse Z transform  $x[n]$ .

a)  $X(z) = \sin(c/z)$

b)  $X(z) = \sin(z/c)$

3) For each of the following systems: list its zeros and poles (and their order); compute its stability; compute the impulse response  $h[n]$ ; compute the Fourier Transform  $H(e^{j\omega})$ ; characterize (and justify your answer) the system as a frequency-selective filter (e.g. high pass, low pass, all pass, band pass, etc.).

a)  $H(z) = \frac{1}{2}z + 1 + \frac{1}{2}z^{-1}$

b)  $H(z) = -\frac{1}{2}z + 1 - \frac{1}{2}z^{-1}$

c)  $H(z) = \frac{1}{2}z - \frac{1}{2}z^{-1}$

4)  $X(z)$  has a double zero at  $z = 0$  and two simple poles at  $z = -1/3$  and  $z = 1/5$ . Using the method of partial fractions, calculate  $x[n]$  (up to overall scale), for the following ROCs:

a)  $\text{ROC} = \{z : |z| > 1/3\}$

b)  $\text{ROC} = \{z : 1/5 < |z| < 1/3\}$

c)  $\text{ROC} = \{z : |z| < 1/5\}$

5) For each of the following causal systems, compute the Z transform of the impulse response, its ROC, and its zeros and poles. Which systems are stable, and why? Is the impulse response of each system right-sided, left-sided, both-sided, or finite-duration? Explain.

a)  $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 2x[n] - \frac{1}{6}x[n-1]$

b)  $y[n] + \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n] - \frac{2}{3}x[n-1]$

c)  $y[n] - \frac{3}{4}y[n-1] - \frac{1}{4}y[n-2] = 2x[n] + \frac{4}{3}x[n-1]$

6) For each of the following sequences, compute the Z transform  $X(z)$ .

a)  $x[n] = n \left( \frac{3}{4} \right)^n u[-n]$

b)  $x[n] = -n \left( \frac{3}{4} \right)^{-n} u[n]$

7) For each of the following causal systems, list its zeros and poles (and their order), its ROC and its stability:

a)  $H_1(z) = \frac{\frac{3}{2} - z^{-1}}{1 - \frac{4}{3}z^{-1} + \frac{1}{3}z^{-2}}$

b)  $H_2(z) = \frac{-\frac{1}{2}z^{-1}}{1 - \frac{4}{3}z^{-1} + \frac{1}{3}z^{-2}}$

c)  $H(z) = H_1(z) + H_2(z)$