ENEE 425

Section 0101 Fall 2004

Homework #4

The problems in this set require Matlab. The problem ***descriptions*** *are long, but there is a lot of repetition (for comparison purposes) and a lot of Matlab code that I give you outright, so the solutions themselves are not abnormally long. You must attach a printed version of Matlab code (a Matlab file is preferable to copying and pasting a session, if possible). You may put several problems into a single printout, if the start of each problem is made obvious. Don't forget to answer the verbal questions too.*

This homework is based on Homework 3. As in that homework, an emulated continuous signal is sampled. We will then decimate that discrete signal. Use this code from that problem to create the "continuous" signal, an 64 second long sinusoid with frequency $f_0 = 0.1875$ Hz, and then sample it with a sample time of $T = 1$ second:

```
tmax = 64; % seconds
granularity = 2^(-11);
t = granularity*[0:tmax/granularity-1]; % seconds
xc = cos(2*pi*0.1875*t);
T = 1; % seconds
nFromt = find(abs(t/T-round(t/T))<eps);% only for T = 1/(power of 2)
n = 0:length(nFromt)-1;
N = length(n);
x = xc(nFromt);
omega = 2*pi*n/N; % radians
freq = omega/T/(2*pi); % Hz (!)
X = fft(x);
```

Now we have the discrete sequence $x[n]$ (since we have both `x` and `n`) and a discrete approximation to the periodic function $X\left(e^{j\omega}\right)$ (since we have both `X` and `omega`).

1) *Downsampling/Decimation.* Open a new figure, and in it allow for 9 `subplots` in a 3 x 3 arrangement.

a) In the first subplot, plot $x[n]$, using `stem()` or `bar()`. Make sure you get the correct values along the n axis. Fix the limits using `axis([0 n(end) min(xc) max(xc)])`. Title it using `title()`.

b) In the next subplot, plot $\left|X\left(e^{j\omega}\right)\right|$, which in Matlab is called `abs(X)` and should be plotted as a function of `omega`, using `stem()` or `bar()`. Make sure you get the correct values of ω along the ω axis. Fix the axis limits using `axis([0 omega(end) 0 max(abs(X))])`. Title it using `title()`. Note that inside `title()`, you can use the expression `\omega` and it will be displayed as ω, since the backslash means to interpret the next string of characters as the name of a symbol.*

c) In the next subplot, plot `abs(X)` as a function of `freq`, using `stem()` or `bar()`. Fix the axis limits using `axis([0 freq(end) 0 max(abs(X))])`. Title it using `title()`. What frequencies do the peaks fall at? Interpreting frequencies above the half-way point as negative, where do the negative frequencies fall at? At which frequencies *should* there be peaks?

Next we will decimate $x[n]$ by a factor of $M = 2$.

First create the ideal digital anti-aliasing filter necessary to create $\tilde{x}[n]$ from $x[n]$ and apply it:

```
H = [1,ones(1,N/M/2-1),zeros(1,N-N/M+1),ones(1,N/M/2-1)];
Xtilde = X.*H;
xtilde = real(ifft(Xtilde));
```

d) In the next subplot, plot $\tilde{x}[n]$, using `stem()` or `bar()`. Make sure you get the correct values along the n axis. Fix the limits using `axis([0 n(end) min(xc) max(xc)])`.Title it using `title()`. Does $\tilde{x}[n]$differ from $x[n]$? Should it?

e) In the next subplot, plot $\left|H\left(e^{j\omega}\right)\right|$, which in Matlab is called `abs(H)` and should be plotted as a function of `omega`, using `stem()` or `bar()`. Make sure you get the correct values of ω along the ω axis. Fix the axis limits using `axis([0 omega(end) 0 max(abs(H))])`. Title it using `title()`. What kind of frequency selective filter is `Hd`? What is the filter's cutoff frequency ω_c ? What fraction of π is this, in terms of M?

f) In the next subplot, plot `abs(H)` as a function of `freq`, using `stem()` or `bar()`. Fix the axis limits using `axis([0 freq(end) 0 max(abs(H))])`. Title it using `title()`. What is the filter's cutoff frequency f_c? What fraction of the sampling frequency is this, in terms of M?

g) In Matlab, from $\tilde{x}[n]$, create the *downsampled* sequence $x_d[n]$ and call it `xd`. **Do not use the Matlab function `decimate()` or `downsample()`—perform the downsampling yourself.**

h) How long is the sequence `xd`? Set `Nd = length(xd);` and set `nd = 0:Nd-1;`. Since `Nd` is not equal to `N`, we will consider `xd` a function of `nd` not `n`, and write $x_d[n_d]$.

i) What is the new sampling time T_d in terms of T and M? Set `Td` equal to that value.

j) In the next subplot, plot $x_d[n_d]$, using `stem()` or `bar()`. Make sure you get the correct values along the n_d axis. Fix the limits using `axis([0 nd(end) min(xc) max(xc)])`. Title it using `title()`. Note that inside `title()`, you can use the expression `x_d` and it will be displayed as x_d, since the underscore means interpret the next characters as a subscript.

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* If you are graphically ambitious, try experimenting with title strings such as: `|X(e^{j\omega})|`. For more title strings you can use "TeX" commands, which you can see in the Matlab Help Desk if you search the index for "TeX".

Define two Fourier space variables, ω_d and f_d appropriate for this downsampling.

```
omegad = 2*pi*nd/Nd; % radians
freqd = omegad/Td/(2*pi); % Hz
```

We find the (discretized version of the) Fourier Transform of $x_d[n_d]$ i.e. $X_d\left(e^{j\omega_d}\right)$ by

```
Xd = fft(xd); % note Xd is not the same as xd
```

k) In a subplot, plot $\left|X_d\left(e^{j\omega_d}\right)\right|$, which in Matlab is called `abs(Xd)` and should be plotted as a function of `omegad`, using `stem()` or `bar()`. Make sure you get the correct values of ω_d along the ω_d axis. Fix the axis limits using `axis([0 omegad(end) 0 max(abs(Xd))])`. Title it using `title()`. Inside `title()`, you can use `\omega_d` to display ω_d.

l) In a subplot, plot `abs(Xd)` as a function of `freqd`, using `stem()` or `bar()`. Fix the axis limits using `axis([0 freqd(end) 0 max(abs(Xd))])`. Title it using `title()`.What frequencies do the peaks fall at? Interpreting frequencies above the half-way point as negative, where do the negative frequencies fall at? At which frequencies *should* there be peaks?

m) In words, describe what would have happened if this same sequence $x[n]$ had been decimated with $M = 4$ instead.

n) *Optional (no credit)* Decimate $x[n]$ by $M = 2$, but use the Matlab function `decimate`. You can do this in one line (it filters and downsamples together). Verify that this new $x_d[n_d]$ and $X_d\left(e^{j\omega_d}\right)$ are similar to those above (they need not be identical since `decimate`'s filter is not ideal).

2) *Upsampling/Interpolation*. This problem will take the sequence $x[n]$ above and manually interpolate it as done in class. Open up a figure and allow for 9 subplots total in a 3 x 3 arrangement.

a) In the first three subplots, plot $x[n]$, $X\left(e^{j\omega}\right)$, and $X\left(e^{j\omega}\right)$ again but as a function of frequency f, *exactly* the first 3 subplots of problem (1).

Next we upsample $x[n]$ by $L = 4$. **Do not use the Matlab function `interp()` or `upsample()`.**

b) Create a the expanded (upsampled) sequence $x_e[n]$ in Matlab, and call it `xe`. Recall that its first value, $x_e[0]$ is equal to $x[0]$, followed by $L - 1$ values set equal to zero, then the next value, $x_e[L]$ is equal to $x[1]$, etc.

c) How long is the sequence `xe`? Set `Ni = length(xe);`. and set `ni = 0:Ni-1;`

d) What is the new sampling time T_i in terms of T and L? Set `Ti` equal to that value.

e) In the next subplot, plot $x_e[n_i]$, using `stem()` or `bar()`. Make sure you get the correct values of n_i along the n_i axis. Fix the axis limits using `axis([0 ni(end) min(xc) max(xc)])`. Title it using `title()`.

Define two Fourier space variables, ω_i and f_i appropriate for this upsampling.

```
omegai = 2*pi*ni/Ni; % radians
freqi = omegai/Ti/(2*pi); % Hz
```

We find the (discretized version of the) Fourier Transform of $x_e[n_i]$ i.e. $X_e\left(e^{j\omega_i}\right)$ by

```
Xe = fft(xe); % note Xe is not the same as xe
```

f) In a subplot, plot $\left|X_e\left(e^{j\omega_i}\right)\right|$, which in Matlab is called `abs(Xe)` and should be plotted as a function of `omegai`, using `stem()` or `bar()`. Make sure you get the correct values of ω_i along the ω_i axis. Fix the axis limits using `axis([0 omegai(end) 0 max(abs(Xe))])`. Title it using `title()`. Note that inside `title()`, you can use the expression `\omega_i` and it will be displayed as ω_i.

g) In a subplot, plot `abs(Xe)` as a function of `freqi`, using `stem()` or `bar()`. Fix the axis limits using `axis([0 freqi(end) 0 max(abs(Xe))])`. Title it using `title()`.What frequencies do the peaks fall at? Interpreting frequencies above the half-way point as negative, where do the negative frequencies fall at? At which frequencies *should* there be peaks?

Create the ideal digital anti-aliasing and scaling filter necessary to create $x_i[n_i]$from $x_e[n_i]$ and apply it:

```
H = L*[1,ones(1,N/2-1),zeros(1,Ni-N+1),ones(1,N/2-1)];
Xi = Xe.*H;
xi = real(ifft(Xi));
```

h) What is the role of L in the filter `H`? As a frequency selective filter, what kind is it?

i) In the next subplot, plot $x_i[n_i]$, using `stem()` or `bar()`. Make sure you get the correct values along the n_i axis. Fix the limits with `axis([0 ni(end) min(xc) max(xc)])`. Title it with `title()`.

j) In a subplot, plot $\left|X_i\left(e^{j\omega_i}\right)\right|$, which in Matlab is called `abs(Xi)` and should be plotted as a function of `omegai`, using `stem()` or `bar()`. Make sure you get the correct values of ω_i along the ω_i axis. Fix the axis limits using `axis([0 omegai(end) 0 max(abs(Xi))])`.

k) In a subplot, plot `abs(Xi)` as a function of `freqi`, using `stem()` or `bar()`. Fix the axis limits using `axis([0 freqi(end) 0 max(abs(Xi))])`. Title it using `title()`.What frequencies do the peaks fall at? Interpreting frequencies above the half-way point as negative, where do the negative frequencies fall at? At which frequencies *should* there be peaks?

l) *Optional (no credit)* Interpolate $x[n]$ by $L = 4$, but time use the Matlab function `interp`. You can do this in one line (it upsamples and filters together). Verify that this new $x_i[n_i]$ and $\left|X_i\left(e^{j\omega_i}\right)\right|$ are similar to those above.