ENEE 425

Section 0101 Fall 2004

Homework #2

You must attach a printed version of Matlab code (a Matlab file is preferable to copying and pasting a session, if possible). You may put several problems into a single printout, if the start of each problem is made obvious. Don't forget to answer the questions requiring verbal answers too. For problems 1 – 3, use Matlab.

Consider these complex finite sequences defined on `n = -8:12`; (21 steps, starting at –8 instead of 0 or 1):

```
x  = exp(j*2*pi*n/5);
y1 = x.*(1-0.99*cos(2*pi*n/15));
y2 = x.*(1-0.99*cos(2*pi*n/15)+0.1*randn(size(n)));
```

[Note that `.*` is multiplication element-by-element, as opposed to `*` which would be matrix multiplication.]

1) Consider $x[n]$.
   a) Describe $x[n]$ in words.
   b) Using `bar()` or `stem()`, plot `abs(x)`, `real(x)`, and `imag(x)` in the appropriate range of n. To ease comparison, put the graphs in one figure but in separate subplots. What is the period of each?
   c) Using `bar()` or `stem()`, plot `angle(x)*180/pi` and `unwrap(angle(x))*180/pi` in the appropriate range of n, putting the plots in the same figure but in separate subplots. Is the phase of this signal linear with n?
2) Consider $y_1[n]$.
   a) Describe $y_1[n]$ in words.
   b) Using `bar()` or `stem()`, plot `abs(y1)` in the appropriate range of n. What is the period? Why is it different from all the periods in 1a?
   c) Using `bar()` or `stem()`, plot `angle(y1)*180/pi` and `unwrap(angle(y1))*180/pi` in the appropriate range of n, putting the graphs in the same figure but in separate subplots. Is the phase of this signal linear with n?
3) Consider $y_2[n]$. The Matlab function `randn()` creates a (pseudo-)random signal with Gaussian distribution, which is a simple way to simulate adding noise.
   a) Describe $y_2[n]$ in words.
   b) Using `bar()` or `stem()`, plot `abs(y2)` in the appropriate range of n. What is the approximate period? How does this graph compare to your graph in 2b?
   c) Using `bar()` or `stem()`plot `angle(y2)*180/pi` and `unwrap(angle(y2))*180/pi` in the appropriate range of n, putting the graphs in the same figure but in separate subplots. Is the phase of this signal approximately linear with n? How does this graph compare to your graph in 2c?

4) Consider an audio Compact Disc. Its acoustic sampling frequency, f_s, is 44.1 kHz, and it uses 16 bits to encode each sample.

a) What is its sampling period T?

b) What is the maximum acoustic frequency it can theoretically encode? Real CD players, for which cost and speed are considerations, can rarely reconstruct frequencies this high (due to their non-ideal lowpass filters). How does this maximum frequency compare to the highest frequencies that humans can reach, 20 kHz: significantly lower, close, or significantly higher?

c) When playing a CD through a specific amplifier we can chose a volume setting so that the softest sounds the CD can make are 0 dB. The softest sounds the CD can make are when the digitized numbers oscillate between 0 and 1, i.e. they use only 1 of the 16 allowed bits. Recall that a multiplicative factor of 2 in amplitude results in a 6 dB gain in power. If the CD uses 2 of the 16 allowed bits, the volume would then be twice as high, e.g. 6 dB. Using this simplistic calculation, what is the full dynamic range of a CD that uses all its bits? How does this compare to the full dynamic range of human hearing, which is roughly 120 dB from softest sounds to loudest sounds? (Later in this chapter we will do the calculation more rigorously.)

d) Repeat (a) – (c) for the newer DVD-Audio format, which uses a minimum acoustic sampling frequency $f_s = 96$ kHz and 24 bits to encode each sample.

5) **Do not use Matlab for this problem.** Consider a fictitious Compact Disc standard with an acoustic sampling frequency of $f_s =$ 40 kHz (making the following plots easier). Its sample time is $T = f_s^{-1}$

a) Consider the continuous sound $x_1(t) = \cos(2\pi f_1 t)$, a pure tone at $f_1 =$ 10 kHz.

i) Plot $x_1(t)$ continuously in the range $[-4T, 4T]$. Circle the values at which it is discretized by the CD, i.e. the values at of t which correspond to integral multiples of T.

ii) Plot $|X_1(j2\pi f)|$ for $f = \omega/(2\pi)$ in the range $[-(f_s + f_1), f_s + f_1]$

iii) Plot $|X_{1s}(j2\pi f)|$ in same range, where $x_s(t) = x(t)s(t)$, where $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$ (depict impulse responses by arrows, as in Figure 4.2 a & b in the textbook).

iv) Plot $|X_{1s}(j2\pi f)H(j2\pi f)|$ in same range, where $H(j2\pi f) = T\big(u(f + f_s/2) - u(f - f_s/2)\big)$ is the ideal lowpass filter with cutoff frequency $f_s/2$.

v) What pure tone (or pure tones) does the inverse Fourier Transform of (iv) correspond to, i.e. at what frequency (or frequencies)? Is this the same as f_1 ? Why or why not?

b) Repeat (a), but instead of $x_1(t)$, use $x_2(t) = \cos(2\pi f_2 t)$, a pure tone with frequency $f_2 =$ 30 kHz (audible to dogs and cats but not humans).