

Homework #8

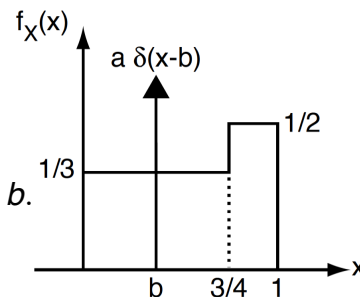
*Remember, the goal here is to understand **how** to do the calculations, not to produce the correct answer using pure instinct. Please explain **how** you have arrived at your answers.*

- 1) Suppose X is an exponential(λ) random variable and $Y = X^2$.
- Calculate $E[Y]$.
 - Find the PDF $f_Y(y)$
- 2) Consider a random variable X that has the following PDF: $f_X(x) = \frac{1}{\pi(x^2 + 1)}$, and suppose $Y = X^2$. Derive

an expression for $f_Y(y)$, the PDF of Y . Hint: you may find it useful to use: $\int \frac{1}{x^2 + 1} dx = \tan^{-1}(x)$.

- 3) Consider a random variable with the PDF illustrated at right.

- Determine the value of the constant a .
- Given that $E[X] = 21/64$, calculate the value of the constant b .



- 4) Let X be a random variable with the following CDF:
$$F_X(x) = \begin{cases} 0 & x < -1 \\ x/4 + 1/2 & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$
- Find $P[X < 1]$ and find $P[X \leq 1]$
 - Calculate the PDF $f_X(x)$.
 - Determine $E[X]$.

- 5) Suppose 20% of a manufacturer's TV sets are made in one manufacturing plant, while the rest are made in another. For those units that come from the first plant, the expected lifetime is described by an exponential distribution with mean of 4 years. For the units from the second plant, the lifetime is exponentially distributed with a mean of 7 years. Let T be the lifetime of a randomly selected TV set.
- Find an expression for the PDF $f_T(t)$. Is T an exponential random variable?
 - Calculate $E[T]$.