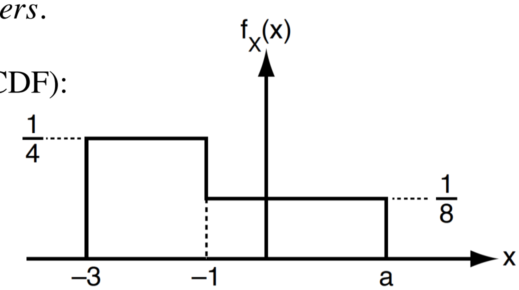
ENEE 324

Sections 0101, 0102, FR01 — Spring 2011

Homework #7

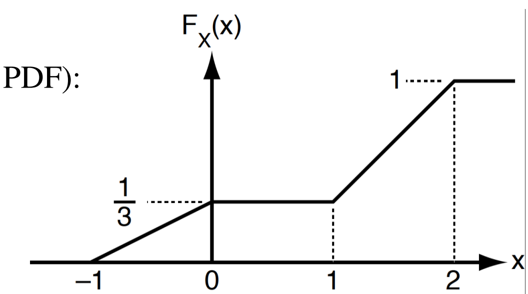
Remember, the goal here is to understand ***how*** *to do the calculations, not to produce the correct answer using pure instinct. Please explain* ***how*** *you have arrive at your answers.*

1) Consider a random variable X that is described by this PDF (*not* CDF):



   a) What is the probability that $X = 0$?
   b) Calculate the value of the constant a.
   c) What is the probability that $|X| > 1$?
   d) Make a clearly labeled sketch of $F_X(x)$, the CDF. You must label all axes, crossings and corners.
   e) Calculate E[X].

2) Consider a random variable X that is described by this CDF (*not* PDF):



   a) Find the PDF $f_X(x)$ for this random variable.
   b) What is $P[1/2 < X < 3/2]$?
   c) Calculate E[X].

3) The Rayleigh probability density function is given $f_X(x) = c\,x \exp\left(-x^2/2b^2\right)u(x)$, where $u(x)$ is the unit step function.
   a) Determine the value of the constant c (express your answer in terms of b).
   b) Find the value of x for which $f_X(x)$ is maximum, i.e. the *mode* (most likely value) of the probability density function.
   c) Find an expression for the cumulative distribution function $F_X(x)$. Make a sketch of this function.
   d) Compute $E[X]$. Hint: You may find the following definite integral useful: $\int_0^\infty \sqrt{u}\, \mathrm{e}^{-u}\, du = \sqrt{\pi}/2$.

4) John Cook, while exploring a remote tropical island encounters a very large chest filled with gold. It is so large in fact, that it will require three persons to carry it back to the ship (him and two others). Suppose that sailors arrive at the treasure site randomly in a Poisson manner with an average rate of $\lambda = 1$ sailors per hour.

a) Let N be a discrete random variable that describes the number of sailors who have arrived to join John Cook after he has been waiting for t hours (where t need not be an integer). What is the probability mass function for N? (Your answer should of course depend upon t.)

b) What is the probability that after waiting for t hours, John Cook still hasn't found two additional people to help him carry the treasure?

c) Now let T be a continuous random variable describing the time that John Cook must wait until he has *enough people to move the treasure*. Use the result from (b) to calculate the cumulative distribution function $F_T(t)$.

d) Use the result from (c) to find $f_T(t)$, the probability density function for T.

e) Find $E[T]$ and $VAR[T]$. Hint: You should recognize the probability density function derived in (d).

5) Let X be a Laplace(λ) random variable, defined by having a PDF of $f_X(x) = \frac{1}{2}\lambda e^{-\lambda|x|}$ (for positive λ).

a) Verify that that this PDF satisfies the probability normalization condition (i.e. total probability is 1)

b) Determine $E[X]$ and $VAR[X]$.