ENEE 324

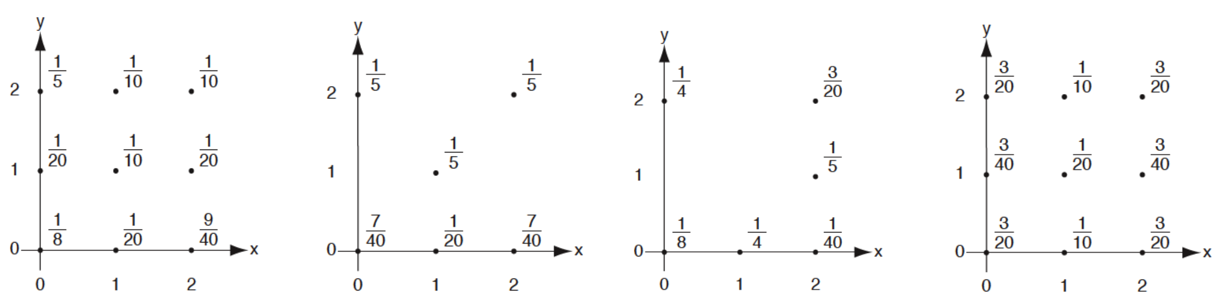
Sections 0101, 0102, FR01 Spring 2011

Homework #6

Remember, the goal here is to understand ***how*** *to do the calculations, not to produce the correct answer using pure instinct. Please explain* ***how*** *you have arrive at your answers.*

1) In each year, a volleyball team plays a total of 3 matches. In each match, there is a 2/3 probability that the team will win, independent of all other matches. The first two matches in the year are played on the home court, while the last match is an "away" game. Let X be the total number of matches won, and let Y be the number of home games won.
   a) Make a tree diagram for this problem; label the leaves with the corresponding values of X and Y.
   b) Calculate and make a labeled sketch of the joint PMF $P_{X,Y}(x,y)$.
   c) What is the marginal PMF of X, $P_X(x)$?
   d) Calculate $COV[X,Y]$ and $E[XY]$.
   e) Determine the conditional PMF of Y, given $X = 2$, and evaluate $E[Y \mid X = 2]$.

(i) (ii) (iii) (iv)



2) For each of the joint PMFs depicted above in (i). . . (iv), determine
   a) whether X and Y are independent
   b) whether $E[X|Y] = E[X]$
   c) whether X and Y are uncorrelated

3) Photons arrive at a photomultiplier tube in a Poisson manner (see note below) with rate λ. That is, the expected number of photons is the product of λ and the time interval over which they are counted. Let X be a random variable that describes the number of photons which are counted in a time interval from 0 to $T/3$, and let Y be a random variable that describes the number of photons that are counted in the time interval from $T/3$ to T. Finally, let W be the total number of photons counted in the whole time interval from 0 to T, i.e., $W = X + Y$. That is: $X \sim$ Poisson($\lambda T/3$), $Y \sim$ Poisson($2\lambda T/3$), and $W \sim$ Poisson(λT).

Note: For a Poisson process, the numbers of counts that occur in non-overlapping time intervals are independent!

a) Find a formula for $P_{X,Y}(x,y)$, the joint PMF of X and Y.

b) Find $P_{X|W}(x\,|\,w)$, the conditional PMF of X given $W = w$.

c) Find the conditional mean and conditional variance of X given $W = w$. Hint: if you obtained the correct answer for (b), this shouldn't be difficult.

4) Xavier and Yolanda are playing a game in which Xavier rolls a fair six-sided die and Yolanda must continue rolling her fair six-sided die until she rolls a number that is greater than or equal to what Xavier obtained. For this problem, consider two random variables X and Y defined as:

X = Outcome from Xavier's roll of die; Y = Number of times Yolanda must roll her die

a) Find the conditional PMF for Y, given that the Xavier rolled a 5, $P_{Y|X}(y\,|\,5)$.

b) Generalize the above result to find an equation for $P_{Y|X}(y\,|\,x)$, the conditional PMF of Y, given that Xavier rolled x.

c) Use the result from parts (a) to determine the probability that ($X = 5$) and ($Y = 2$).

d) Use the result from (b) to find a formula that describes $P_{X,Y}(x,y)$.

e) Use the law of iterated expectation to calculate E[Y].

5) An embedded system design employs 8 interrupt lines, numbered 1-8. Each of these lines can be assert any of 16 different software signal codes, the integers from 1-16. Interrupt priority is determined both by the numerical values of both the software signal code and the hardware interrupt line. Specifically, the "hardware interrupt priority" is the number of the interrupt line , the "software interrupt priority" is the value of the signal code, and the "total interrupt priority" is the sum of the two (e.g. signal 2 on line 1 is only a total interrupt priority of 3, but signal 8 on line 4 is a total interrupt priority of 12). The probability of any given software signal on any interrupt line is determined empirically to be inversely proportional to the total interrupt priority. Using Matlab (e.g. with the `ndgrid()` function and others), determine

a) the joint probability mass function

b) the minimum, maximum, and expectation value of the *hardware* interrupt priority

c) the minimum, maximum, and the expectation value of the *software* interrupt priority

d) the minimum, maximum, and the expectation value of the *total* interrupt priority.

Please attach all your Matlab code.