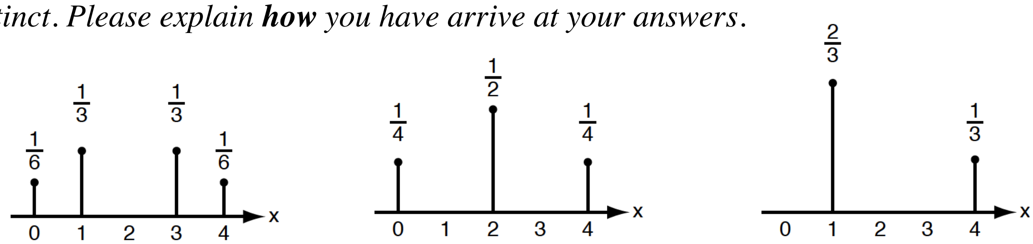
ENEE 324

Sections 0101, 0102, FR01 Spring 2011

Homework #5

Remember, the goal here is to understand ***how*** *to do the calculations, not to produce the correct answer using pure instinct. Please explain* ***how*** *you have arrive at your answers.*



(b) (c) (d)

1) In the last homework, for each of the discrete random variables with the PMFs above, (b-d), you calculated the mean μ_X and standard deviation σ_X. Now use Matlab to generate 600 samples for each of the three random variables. For each set of samples, use Matlab to: (i) display their histograms, (ii) calculate their mean and standard deviation (use `mean()` and `std()`), and (iii) describe in words how the results of (i) and (ii) agree with the analytic results calculated previously. *Please attach all your Matlab code.*

2) Suppose you roll two fair six-sided dice and observe the outcome of each roll. Consider two random variables, X and Y, defined as: X = # of dots facing up on first die; Y = # of dots facing up on second die.
   a) Find $P[X = Y]$
   b) Find $P[|X-Y| \leq 3]$
   c) Find $P[XY \leq 5]$
   d) Find $P[X+Y \geq 5]$

   Now consider a new random variable W defined as $W = X - Y$.
   e) Find $P_W(w)$, the PMF of W.
   f) Find $E[W]$ and $VAR[W]$.

3) Consider an experiment in which you test two circuits. The probability that either of the circuits is good is 0.95, independent of the other circuit. Define two random variables X and Y as follows:
   X = total number of good circuits found; Y= {1, if the if the second circuit was good, but 0 otherwise}
   a) Draw a labeled tree diagram for these two independent trials. Label the leaves of the tree diagram with the corresponding values for X and Y.
   b) Use the tree diagram to find the joint PMF of X and Y, $P_{X,Y}(x,y)$. Express your answer as a series of points in the (x, y) plane with labels indicating the probability of each point.

c) Find the mean and variance of X.

d) Find the mean and variance of Y.

e) Find the correlation, $E[XY]$.

f) Find the covariance, $COV[X, Y]$.

4) In problem 4 from Homework 4, we described a grading policy in which Prof. Arbitrarius randomly assigned grades by rolling two 5-sided dice each labeled with the grades 0 to 4, and using the smaller of the two outcomes for the student's grade. Consider three random variables X, Y, and G, defined as: X = outcome of first die; Y = outcome of second die; $G = \min(X, Y)$ = student's final grade

a) What is the joint PMF of X and Y, $P_{X,Y}(x,y)$?

b) What is the probability that $G = 1$, given that $X = 2$?

c) What is the probability that $X = 2$ and $G = 1$? (Note that this will correspond to $P_{X,G}(2,1)$.)

d) Compute the joint PMF of X and G, $P_{X,G}(x,g)$. You may express your answer either as a series of points in the (x, g) plane with labels indicating the probability of each point, or you may write a table that expresses the probability of each (x, g) combination.

e) Find the marginal PMF $P_G(g)$, and verify that it agrees with the solution from the last problem set.

f) Compute the covariance $COV[X, G]$.

5) Prove the following properties of covariance:

a) $COV[X,Y + b] = COV[X,Y]$

b) $COV[aX,Y] = a\,COV[X,Y]$.

c) $COV[X,Y + Z] = COV[X,Y] + COV[X,Z]$.

6) Consider the random variables X and Y with the joint PMF: $P_{XY}(x,y) = \begin{cases} c|x+y| & x = -1,0,1, \quad y = -2,0,2 \\ 0 & \text{otherwise} \end{cases}$

a) What is the value of the constant c?

b) Find the marginal PMFs, $P_X(x)$ and $P_Y(y)$

c) Find the expected values $E[X]$ and $E[Y]$.

d) Find the variances $VAR[X]$ and $VAR[Y]$.

e) Calculate $E[XY]$ and $COV[X,Y]$.