# ENEE 324

Sections 0101, 0102, FR01 Spring 2011

**Homework #4**

*Remember, the goal here is to understand* ***how*** *to do the calculations, not to produce the correct answer using pure instinct. Please explain* ***how*** *you have arrive at your answers.*

1) The probability that a particular kind of light bulb will burn out during its $K$th month of use is described by the PMF: $P_K(k) = 0.1(0.9)^{k-1}$.
   a) Find $\mu_K = E[K]$, the average time to failure (in months) for such a bulb.
   b) Find $\sigma_K^2 = VAR[K]$, the variance of the time to failure (in months) for such a bulb. What are its units?
   c) Find $\sigma_K$, the standard deviation of the time to failure (in months) for such a bulb. What are its units?
   d) Use the results of (a) and (c) to come up with a simple, written explanation/prediction, suitable for a non-engineer, of a time frame over which to expect when this bulb will burn out.
   e) In answering (d), why is the answer to (c) much more immediately suited than the answer to (b)?

2) Consider a bag that contains two black marbles and three white marbles. You draw two marbles from the bag, one at a time. Whenever you draw a black marble, you keep it. If, however, the first marble drawn is a white marble, then you throw it back into the bag before drawing the second one. Let $X$ be a random variable that describes the total number of black marbles you are holding at the end.
   a) What is the range of $X$?
   b) Make a clearly-labeled sketch of $P_X(x)$, the probability mass function (PMF) of $X$.
   c) Evaluate $E[X]$ and $VAR[X]$.
   d) Make a clearly-labeled sketch of $F_X(x)$, the cumulative distribution function (CDF) of $X$.
   e) Evaluate $P[X > \frac{2}{3}]$.

3) Suppose you go to the local store to replace a blown fuse in your home. The fuses are packaged individually, and the likelihood that any given fuse is defective is 1/5, independent of the others. Let $U$ be a random variable describing the number of fuses that you must purchase in order to get one good one. Then the store manager offers you a deal: the cost per fuse decreases according to $C(U) = 1 + (1/4)^{U-1}$, where $U$ is the number of fuses that you wish to purchase and $C$ is the cost per fuse in dollars (i.e., the cost per fuse drops asymptotically to \$1 per fuse when you buy lots of them).
   a) Find the PMF of $U$.
   b) Find the mean and variance of $U$.
   c) Find the expected total cost $T$ of finding a working replacement fuse.

4) Professor Arbitrarius uses an unconventional method to assign grades for his students. He rolls a pair of fair five-sided dice that are each labeled with the grades 0 to 4 (corresponding to grades F, D, C, B, & A). Then he uses whichever die has the lower result as the student's grade. Let $G$ be a random variable describing the numerical value of the student's final grade for this class.

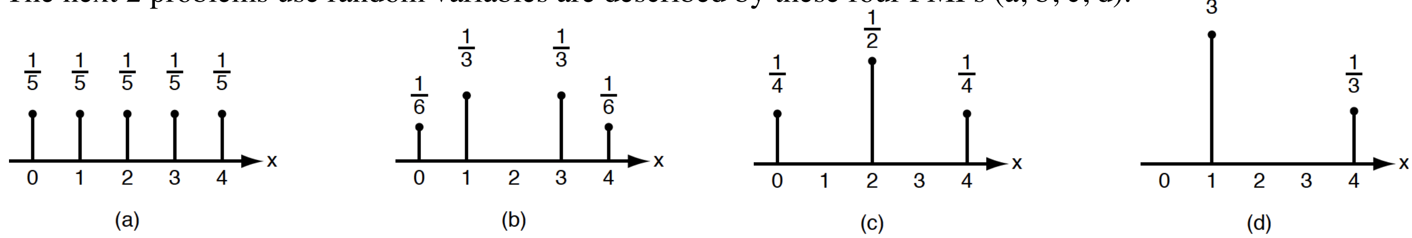
a) What is the range $S_G$ of the random variable $G$?

b) Determine $P_G(g)$, the probability mass function (PMF) of $G$. Hint: you may wish to make a table of all possible dice roll outcomes.

c) Find the probability that a student will score a grade "B" or better in the course (i.e., 3 or higher).

d) Compute the mean and standard deviation of $G$.

The next 2 problems use random variables are described by these four PMFs (a, b, c, d):




5) For each of the discrete random variables given above, calculate the mean $\mu_X$ and standard deviation $\sigma_X$.

6) For the first only(!) discrete random variable given above, (a), use Matlab to generate 1000 samples. For those samples: (i) display their histograms, (ii) calculate their mean and standard deviation (use `mean()` and `std()`), and (iii) in words describe how the results of (i) and (ii) agree with the results of the previous problem. *Please attach all your Matlab code.*

7) *Bonus (just for fun, no extra credit)*: When Joe eventually falls sleep at night, he is always lying on his back. However, during the night he flips over onto his belly, and then back onto his back, etc., at random times. Suppose the number of flips (the kind of flip doesn't matter) is described by a Poisson random variable with mean $\alpha$. Let $X$ be a random variable that describes how many flips he executes in one night, i.e. $X \sim \text{Poisson}(\alpha)$.

a) In terms of $\alpha$, find an expression for the $P[B]$, the probability that Joe will wake up lying on his back.

b) Find an expression for the conditional probability mass function for $X$ given that he wakes up on his back, in terms of $\alpha$.

c) Compute the conditional expected value of $X$ given that Joe wakes up on his back, in terms of $\alpha$.

Please use these identities to simplify: $\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$ $\quad \sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$