ENEE 324

Sections 0101, 0102, FR01 Spring 2011

Homework #2

Remember, the goal here is to understand ***how*** *to do the calculations, not to produce the correct answer using pure instinct. Please explain* ***how*** *you have arrive at your answers.*

1) Consider two different random experiments.
   a) A particular random experiment is defined by tossing two six-sided dice and noting the magnitude of the difference of the two numbers.
      i) What is the sample space S_1?
      ii) Write out the set *A* corresponding to the event "the magnitude of the difference is 4".
   b) Another random experiment is defined by tossing two six-sided dice and noting the sum.
      i) What is the sample space S_2?
      ii) Write out set *B* corresponding to the event "the sum is 4".
   c) Does it make sense to ask what is $A \cup B$? Why or why not?

2) Consider two random numbers (x,y) drawn with equally likely probability between 0 and 1. What is the probability that their product, xy, is less than e^{-1} (e is Euler's constant)? You might find it helpful to draw the curve $xy = \mathrm{e}^{-1}$ restricted to the unit square.

3) Use this "Probability of Death, by Age Group" table from the National Center for Health Statistics (from 50 years ago). Compute the probabilities:
   a) that a person who was then 30 years old would die before reaching their 40th birthday. Hint: the reduced outcome space for this conditional probability is defined by the event that the person has not yet died before reaching 30 years old.
   b) that a 10 year old would die before reaching 20.
   c) that a 50 year old will live at least one more decade.

Age (years)	Prob. of death
0-10	3.23%
10-20	0.65%
20-30	1.21%
30-40	1.84%
40-50	4.31%
50-60	9.69%
60-70	18.21%
70-80	27.28%
80 & over	33.58%
Total	100%

4) Suppose you have a bag containing 5 coins. Three of the coins in the bag are fair coins (result in heads or tails with equal probability). The remaining two coins in the bag are biased towards heads such that they come up heads with probability 2/3 when flipped. Consider an experiment in which you randomly select

a coin from the bag and then flip it. Let F denote the event that the coin you selected is fair and let H denote the event that the outcome of the flip is heads.

a) Draw a fully labeled tree diagram for this experiment. Use the tree diagram to compute $P[H]$ the probability that the outcome is heads.

b) Given that the outcome is heads, what is the probability that the coin is fair?

c) Suppose you wish to make an educated guess whether the selected coin is fair or biased based upon the observed outcome. Based on your answer to (b), you naively decide to guess that the coin is fair if the outcome is heads and biased if the outcome is tails. What is the probability that your guess will be correct? Hint: identify which tree leaves diagram correspond to correct guesses for this strategy.

d) Bonus (just for fun, no extra credit): Find a better strategy for guessing whether the coin is fair or biased based upon the same observations.

5) Consider a portable CD player that uses 3 AA batteries. One day, it stops working because all of the batteries died. You buy an 8-pack of replacement batteries, and put them in a pile. While removing the *old* batteries, you accidentally mix 2 of the dead batteries with the 8 new ones, for a total of 10.

a) Suppose you randomly pick 3 batteries. What is the probability that none of those batteries are dead?

b) Suppose that the CD player only requires that two or more of its three batteries be good. What is the probability that the unit will work if you install three randomly selected batteries from the pile? Hint: You might start with the number of possible selections of exactly two good batteries & one dead one.

c) Given that the CD player works, what is the probability that you have installed three good batteries instead of two good ones and one dead one?

6) In the last homework, you were given the following information: In Virginia, you are allowed to drive your car in the HOV lane if you are carrying at least two passengers (T) or if you are driving a fuel-efficient hybrid vehicle (H). Suppose a survey reports that 20% of the cars in the HOV lanes are hybrid vehicles, and 90% of the cars in the HOV lane are carrying at least two passengers. Furthermore, police records report that 5% of the cars in the HOV lane are violating the law.

a) Suppose a police officer randomly stops a car in the HOV lane. Given that the car is not a fuel-efficient hybrid vehicle, what is the probability that the officer will issue a ticket?

b) Given that the officer did not issue a ticket, what is the probability that the car is a hybrid vehicle?

c) Suppose you randomly select a hybrid vehicle in the HOV lane. What is the likelihood that the car will have only one passenger?

d) Are the events H and T independent? Explain.