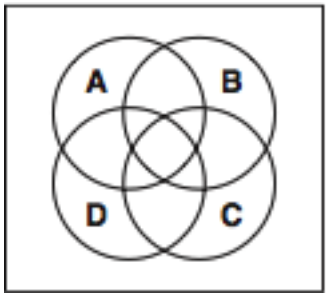
ENEE 324

Sections 0101, 0102, FR01 Spring 2011

Homework #1

*Remember, the goal here is to understand **how** to do the calculations, not to produce the correct answer using pure instinct. Please explain **how** you have arrive at your answers.*

1) Venn diagrams can be useful tools for understanding set manipulations. They may not *fully* describe situations containing **more** than 3 sets (or events), however, as we will show in this problem.
   a) Draw three Venn diagrams for problems involving 1, 2, and 3 events, respectively. See (c) below as an example of the 4 event case.
   b) Count the number of distinct (non-overlapping) regions in each of the Venn diagrams that you drew in part (a). Hint 1: In the figure below there are 14 distinct regions. Hint 2: Note that even the zero event case still has one region (S).
   c) Generalize the answers above for 1, 2, and 3 events to the case of n events. How many distinct regions should there be? Hint: ignore the figure below.
   d) The following figure is an attempt to draw a Venn diagram for a problem involving 4 events, A, B, C and D. Show that this Venn diagram is **not** complete by *finding an event that does not correspond to any region in the Venn diagram*. Express your answer in terms of an intersection the events A, B, C and D and their complements. This proves that the answer to (c) for the case of $n = 4$ is *not* 14.



2) Consider three events A, B and C. For each of the following compound events below: (i) draw a Venn diagram, shading the region corresponding to the compound event described, and (ii) express the compound event as a logical combination (using unions, intersections and complements) of the events A, B and C. (For brevity, the $\cap$ may be omitted when expressing an intersection, e.g., you may write $A \cap B$ as AB.)
   a) Either A or B, but not C
   b) A, but neither B nor C
   c) Exactly one of the events A, B and C.
   d) No more than one of the events A, B and C.
   e) At least two of the events A, B and C.
   f) At most two of the events A, B and C.

3) Which (if any) of the following events are the same, and explain your reasoning:
   a) $AB^C C^C \cup ABC^C \cup ABC \cup A^C BC \cup A^C BC^C$
   b) $B \cup AC^C$
   c) $C^C \cap (A \cup B)$
   d) $BC \cup AB \cup B \cup AC^C$
   e) $(C^C \cap (A \cup B)) \cup (B \cap C)$
   f) $(A \cup B) \cap (B \cup C^C)$

4) Use the axioms of probability to prove that:
   a) $P\left[A \cap B^c\right] = P[A] - P[A \cap B]$
   b) $P\left[A^c \cap B^c\right] = 1 - P[A] - P[B] + P[A \cap B]$

5) In Virginia, you are allowed to drive your car in the HOV lane if you are carrying at least two passengers (T) **or** if you are driving a fuel-efficient hybrid vehicle (H). Suppose a survey reports that 20% of the cars in the HOV lanes are hybrid vehicles, and 90% of the cars in the HOV are carrying at least two passengers. Furthermore, police records report that 5% of the cars in the HOV lane are violating the law.
   a) What is the probability that a car randomly selected from the HOV lane will have only one passenger?
   b) What is the probability that a randomly selected car in the HOV lane will be a hybrid vehicle carrying at least two passengers?