ENEE 322

Sections 0101 & 0102 Spring 2017

Homework #14

1) Consider a fictitious audio standard that has an acoustic sampling frequency of $f_s = 50$ kHz (this will make the following plots easier). Its sample time is $T_s = f_s^{-1}$.

a) Consider the continuous sound $x_1(t) = \cos(2\pi f_1 t)$, a pure tone at $f_1 = 20$ kHz.

i) Plot $x_1(t)$ continuously in the range $[-5T_s, 5T_s]$. Circle the values at which it is discretized by

the CD, i.e. the values at of *t* which correspond to integral multiples of T_s.

ii) Plot $|X_1(j2\pi f)|$ for $f = \omega/(2\pi)$ in the range $[-(f_s + f_1), f_s + f_1]$.

iii) Plot $|X_1^p(j2\pi f)|$ in same range, where $x^p(t) = x(t)p(t)$, where $p(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT_s)$ (depict impulse responses by arrows, as in the textbook's figure 7.3).

iv) Plot $|X_1^p(j2\pi f)H(j2\pi f)|$ in same range, where $H(j2\pi f) = T_s\left(u(f + f_s/2) - u(f - f_s/2)\right)$ is the ideal lowpass filter with cutoff frequency $f_s/2$.

v) What pure tone (or pure tones) does the inverse Fourier Transform of (d) correspond to, i.e. at what frequency (or frequencies)? Is this the same frequency as the original?

b) What would happen if instead of $x_1(t)$ with frequency f_1, you used $x_2(t)$ a pure tone with frequency $f_2 = 40$ kHz (audible to cats and other animals, but not us)?

2) For each of the following signals, **using the definition of the Z transform**, compute the Z transform, its region of convergence (ROC), its the zeros and poles. Sketch the ROC and the locations of zeros and poles. Check also for zeros and poles at $|z| = \infty$ (i.e. check to see if $\lim_{|z|=\infty} X(z) = 0$ or ∞).

a) $x[n] = u[n]$

b) $x[n] = u[-n]$

c) $x[n] = u[-n-1]$

3) For each of the following signals, compute the Z transform, its ROC, its the zeros and poles. Sketch the ROC and the locations of zeros and poles. Check also for zeros and poles at $|z| = \infty$. Assume $N = 6$ and $a > 0$ for the sketches.

a) $x[n] = u[n]\sin\left(\frac{2\pi}{N}n\right)$

b) $x[n] = a^{|n|}$

4) For each of the following signals, compute the Z transform, its ROC, its the zeros and poles. Sketch the ROC and the locations of zeros and poles. Check also for zeros and poles at $|z| = \infty$. If the Z transform does not exist, (i.e. the ROC is the null set), point this out. Assume $b > 0$ for the sketches.

a) $x[n] = u[n]b^n + u[n](3b)^n$

b) $x[n] = (u[n+3] - u[n])b^n$

5) For each of the following causal systems, compute the Z transform, its ROC, and its zeros and poles. Which systems are stable, and why?

a) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n] - \frac{3}{4}x[n-1]$

b) $y[n] - \frac{3}{2}y[n-1] + \frac{1}{2}y[n-2] = 2x[n] - \frac{3}{2}x[n-1]$

c) $y[n] - 3y[n-1] + 2y[n-2] = 2x[n] - 3x[n-1]$