

Homework #13

- 1) For the following causal systems, use the Laplace transform to demonstrate which are stable.
- $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 5x(t) + 2\dot{x}(t)$
 - $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 3x(t) + 2\dot{x}(t)$
 - $\ddot{y}(t) + \dot{y}(t) = x(t) + 2\dot{x}(t)$
- 2) Consider a fictitious audio standard that has an acoustic sampling frequency of $f_s = 50$ kHz (this will make the following plots easier). Its sample time is $T_s = f_s^{-1}$.
- Consider the continuous sound $x_1(t) = \cos(2\pi f_1 t)$, a pure tone at $f_1 = 20$ kHz.
 - Plot $x_1(t)$ continuously in the range $[-5T_s, 5T_s]$. Circle the values at which it is discretized by the CD, i.e. the values at of t which correspond to integral multiples of T_s .
 - Plot $|X_1(j2\pi f)|$ for $f = \omega/(2\pi)$ in the range $[-(f_s + f_1), f_s + f_1]$.
 - Plot $|X_1^p(j2\pi f)|$ in same range, where $x^p(t) = x(t)p(t)$, where $p(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT_s)$ (depict impulse responses by arrows, as in the textbook's figure 7.3).
 - Plot $|X_1^p(j2\pi f)H(j2\pi f)|$ in same range, where $H(j2\pi f) = T_s(u(f + f_s/2) - u(f - f_s/2))$ is the ideal lowpass filter with cutoff frequency $f_s/2$.
 - What pure tone (or pure tones) does the inverse Fourier Transform of (d) correspond to, i.e. at what frequency (or frequencies)? Is this the same frequency as the original?
 - What would happen if instead of $x_1(t)$ with frequency f_1 , you used $x_2(t)$ a pure tone with frequency $f_2 = 40$ kHz (audible to cats and other animals, but not us)?
- 3) For each of the following signals, **using the definition of the Z transform**, compute the Z transform, its region of convergence (ROC), its zeros and poles. Sketch the ROC and the locations of zeros and poles. Check also for zeros and poles at $|z| = \infty$ (i.e. check to see if $\lim_{|z| \rightarrow \infty} X(z) = 0$ or ∞).
- $x[n] = u[n]$
 - $x[n] = u[-n]$
 - $x[n] = u[-n - 1]$

d) $x[n] = u[n] \sin\left(\frac{2\pi}{N}n\right)$ (for the sketch only assume $N = 6$)

e) $x[n] = a^{|n|}$ (for the sketch only assume $a > 0$)

4) $X(z)$ has a double zero at $z = 0$ and two simple poles at $z = -1/2$ and $z = 1/3$. Using the method of partial fractions, calculate $x[n]$ (up to overall scale), for the following ROCs:

a) ROC = $\{z: |z| > 1/2\}$

b) ROC = $\{z: 1/3 < |z| < 1/2\}$

c) ROC = $\{z: |z| < 1/3\}$

5) For each of the following causal systems, compute the Z transform, its ROC, and its zeros and poles.

Which systems are stable, and why?

a) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n] - \frac{3}{4}x[n-1]$

b) $y[n] - \frac{3}{2}y[n-1] + \frac{1}{2}y[n-2] = 2x[n] - \frac{3}{2}x[n-1]$

c) $y[n] - 3y[n-1] + 2y[n-2] = 2x[n] - 3x[n-1]$