ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #11

1) Consider the causal filter defined by the differential equation $\dot{y}(t) = -3y + x/2$:
   a) Compute (and simplify) the magnitude $|H(j\omega)|$
   b) Compute (and simplify) the phase $\Phi_H(\omega)$
   c) Characterize this filter as one of {Low pass, Band pass, High pass, All pass, none of the above} and justify your answer.
   d) Graph a Bode plot of this transfer function. Plot both amplitude and phase, with amplitude in dB and phase in radians, and with ω on a logarithmic scale, centered on $\omega = 1$, showing a few decades on either side. You are welcome to use a computer (e.g. Matlab).
   e) In the region where $|H(j\omega)|$ is falling off, how fast does it fall off in dB/decade?
   f) Compute the group delay $\tau(\omega)$ and simplify.
   g) Calculate the group delay $\tau(\omega)$ at $\omega = 0$ and for $\omega >> 1$
   h) Compute the step response $s(t)$ (i.e. the response to a step input $u(t)$)
   i) Compute the final value of the step response, $s(\infty)$
   j) Calculate the overshoot of $s(t)$.
   k) Calculate at what value of *t* will $s(t)$ be within 1% of $s(\infty)$.

2) Consider the transfer function $H(j\omega) = \dfrac{1 - j\omega/2}{1 + j\omega/2}$:
   a) Compute (and simplify) the magnitude $|H(j\omega)|$
   b) Compute (and simplify) the phase $\Phi_H(\omega)$
   c) Characterize this filter as one of {Low pass, Band pass, High pass, All pass, none of the above} and justify your answer.
   d) Graph a Bode plot of this transfer function. Plot both amplitude and phase, with amplitude in dB and phase in radians, and with ω on a logarithmic scale. You are welcome to use a computer.
   e) Compute the group delay $\tau(\omega)$ and simplify. You will probably need $\dfrac{d}{dx}\tan^{-1}(x) = \dfrac{1}{1+x^2}$.
   f) Calculate the group delay $\tau(\omega)$ specifically at $\omega = 0$ and for $\omega >> 1$.

3) Consider the LTI system $y(t) = 4\dot{x}(t)$:

   a) Compute (and simplify) the impulse response $h(t)$

   b) Compute (and simplify) the transfer function $H(j\omega)$

   c) Compute (and simplify) the magnitude $|H(j\omega)|$

   d) Compute (and simplify) the phase $\Phi_H(\omega)$

   e) Compute the group delay $\tau(\omega)$ and simplify.

   f) Plot $|H(j\omega)|$ using linear amplitude and linear ω, for $\omega > 0$.

   g) Graph a Bode plot of this transfer function. Plot both amplitude and phase, with amplitude in dB and phase in radians, and with ω on a logarithmic scale.

   h) In the region where $|H(j\omega)|$ is rising, how fast does it rise in dB/decade?

4) Consider the discrete system $y[n] = -x[n]/4 + x[n-1]/2 - x[n-2]/4$:

   a) Compute (and simplify) the impulse response $h[n]$.

   b) Plot the step response $s[n]$ (i.e. the response to a step input $u[n]$), and $u[n]$ on the same graph.

   c) Compute (and simplify) the transfer function $H(e^{j\omega})$.

   d) Compute (and simplify) the magnitude $|H(e^{j\omega})|$. Characterize this filter as one of {Low pass, Band pass, High pass, All pass, none of the above} and justify your answer.

   e) Compute (and simplify) the phase $\angle H(e^{j\omega})$. Is it continuous (up to jumps of 2π)?

   f) Graph a Bode plot of this transfer function (with ω from 0 to π). Plot both amplitude and phase, with amplitude in dB and phase in radians.