ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #10

1) Consider the following discrete signals:

$x_1[n] = u[n] - u[n-1] = \delta\left[n\right]$

$x_2[n] = u[n] - u[n-2] = \delta\left[n\right] + \delta\left[n-1\right]$

$x_{N_1}[n] = u[n] - u[n-N_1] = \delta\left[n\right] + \delta\left[n-1\right] + \cdots + \delta\left[n-(N_1-1)\right]$

a) Plot $x_1[n]$

b) Plot $x_3[n]$ (i.e., using $N_1 = 3$).

c) Compute the Fourier Transform $X_1(\mathrm{e}^{j\omega})$ of $x_1[n]$

d) Compute the Fourier Transform $X_2(\mathrm{e}^{j\omega})$ of $x_2[n]$

e) Compute the Fourier Transform $X_{N_1}(\mathrm{e}^{j\omega})$ of $x_{N_1}[n]$

2) Consider the following discrete, periodic functions (what is their fundamental period?):

$$x_1[n] = \sum_{l=-\infty}^{\infty} \left(u[n-lN] - u[n-1-lN]\right) = \sum_{l=-\infty}^{\infty} \delta[n-lN]$$

$$x_2[n] = \sum_{l=-\infty}^{\infty} \left(u[n-lN] - u[n-2-lN]\right) = \sum_{l=-\infty}^{\infty} \left(\delta[n-lN] + \delta\left[n-1-lN\right]\right)$$

$$x_{N_1}[n] = \sum_{l=-\infty}^{\infty} \left(u[n-lN] - u[n-N_1-lN]\right) = \sum_{l=-\infty}^{\infty} \left(\delta[n-lN] + \cdots + \delta\left[n-(N_1-1)-lN\right]\right) \text{ for } N_1 < N .$$

a) Plot $x_2[n]$ from $[-N, 2N]$ for $N = 6$

b) Compute the Fourier Series a_k of $x_2[n]$ for $N = 6$

c) Compute the Fourier Series a_k of $x_{N_1}[n]$ for general N.

d) Compute the Fourier Transform $X(\mathrm{e}^{j\omega})$ of $x_2[n]$ for general N.

e) Compute the Fourier Transform $X(\mathrm{e}^{j\omega})$ of $x_{N_1}[n]$ for general N.

3) Compute and simplify the Fourier Transforms of the following expressions, paying particular attention to the DC component, and describing the behavior of $X(\mathrm{e}^{j\omega})$ near $\omega = 0$.

a) $u[n] - \frac{1}{2}$

b) $u[n]+\frac{1}{2}$

c) $1-u[n]$

d) $\frac{1}{2}-u[n]$

e) $u[-n]$

f) $u[-n+2]$

4) Compute the Fourier Transforms of the following expressions and simplify

a) $\mathrm{e}^{-an}\,u[n]$ for $a>0$

b) $n\,\mathrm{e}^{-an}\,u[n]$ for $a>0$

c) $(n+1)\mathrm{e}^{-an}\,u[n]$ for $a>0$ (This simplifies to an expression useful in the next problem)

d) $\mathrm{Diff}\left\{\mathrm{e}^{-an}\,u[n]\right\}$ for $a>0$

e) $\mathrm{Run}_{-\infty}\left\{\mathrm{e}^{-an}\,u[n]\right\}$ for $a>0$

f) $\left(\mathrm{e}^{-an}\,u[n]\right)*\left(\delta[n]-\delta[n-1]\right)$ for $a>0$

g) $\mathrm{e}^{-|an|}$ (feel free to use the identity $f[|s|]=u[n]f(n)+u[-n]f[-n]-f[n]\delta[n]$).

5) Compute and simplify (using partial fractions) the Fourier transform of $\left(\mathrm{e}^{-an}\,u[n]\right)*\left(\mathrm{e}^{-bn}\,u[n]\right)$ for both $a,b>0$, showing your work. Compute the inverse Fourier transform of the result. Do both cases, $a\neq b$ and $a=b$.