ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #9

1) Consider the signal $x(t) = A\ \mathrm{sinc}(4 f_0 t)$:
   a) Sketch $x(t)$ for the range $[-1/f_0, +1/f_0]$.
   b) Compute its Fourier Transform $X(j\omega)$. **Hint** There is a hard way to do it, and an easy way to do it.
   c) Sketch $X(j\omega)$ for the range $[-8\pi f_0, +8\pi f_0]$

2) Compute the Fourier Transform $X(j\omega)$ for:
   a) $x(t) = 3\sin(2\pi t/5)$
   b) $x(t) = 3\sin(2\pi t/5) + 1$

3) For each of these functions, plot it for the range $[-2T_1, +2T_1]$ and compute its Fourier Transform. Use the properties of the Fourier Transform to simplify the calculation whenever possible.
   a) $x_a(t) = t\left[u\left(t + T_1/2\right) - u\left(t - T_1/2\right)\right]$
   b) $x_b(t) = \left(t/2\right)\left[u\left(\left(t/2\right) + T_1/2\right) - u\left(\left(t/2\right) - T_1/2\right)\right]$
   c) $x_c(t) = t\left[u\left(t\right) - u\left(t - T_1\right)\right]$
   d) $x_d(t) = \left(-t\right)\left[u\left(-t\right) - u\left(-t - T_1\right)\right]$
   e) $x_e(t) = t\left[u\left(t + T_1\right) - u\left(t\right)\right]$
   f) $x_f(t) = x_c(t) + x_e(t)$
   g) $x_g(t) = x_c(t + T_1) - x_e(t - T_1)$

4) As done in class and in the text, $x(t) = u(t)\mathrm{e}^{-at}$ has the Fourier Transform $X(j\omega) = 1/\left(a + j\omega\right)$ for $a > 0$.
   a) Calculate the Fourier Transform of $\dfrac{dx}{dt}$, entirely in the Fourier Domain (i.e. starting from $X(j\omega)$).
   b) Calculate $\dfrac{dx}{dt}$ in the time domain.
   c) Calculate the Fourier Transform of $\dfrac{dx}{dt}$ calculated in (b) using the standard equation.

d) Show that the answers to (a) and (c) are equal.

5) The amplitude modulation of a carrier signal with frequency $f_C = \frac{\omega_C}{2\pi}$, by a (positive) signal $x(t)$, is given by $y(t) = x(t)\cos(\omega_C t)$. Calculate its Fourier Transform $Y(j\omega)$, in terms of $X(j\omega)$ the Fourier Transform of $x(t)$. Optional: Interpret the Fourier Transform $Y(j\omega)$ in terms of $x(t)$ or $X(j\omega)$ and ω_C .

6) Consider the following spatial impulse responses $h(s)$. For each case, plot $h(s)$ for the range $[-6/a\ ,\ +6/a]$. Compute its Fourier Transform $H(j\omega)$. Is it low pass, band pass, or high pass? Recall that $f(|s|) = u(s)f(s) + u(-s)f(-s)$ for any signal $f(s)$.

a) $h(s) = \mathrm{e}^{-a|s|}$, for $a > 0$.

b) $h(s) = as\,\mathrm{e}^{-a|s|}$, for $a > 0$.

7) Compute the Fourier Transforms of the following expressions, by any means, and simplify

a) $\cos(\omega_0 t) * u(t)$

b) $\cos(\omega_0 t) * \left(u(t)\mathrm{e}^{-at}\right)$

c) $u(t)\mathrm{e}^{j\omega_0 t}$

d) $\left(u(t)\mathrm{e}^{j\omega_0 t}\right) * \left(u(t)\mathrm{e}^{-at}\right)$