ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #8

Important Attach a hardcopy of all MATLAB code.

1) As done in class, the zero-DC square wave function, with period $T = 1$, is

$x(t) = -\frac{1}{2} + u\left(t + \frac{1}{4}\right) - u\left(t - \frac{1}{4}\right)$ for $|t| < \frac{1}{2}$, extended periodically. Its Fourier expansion,

$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{2\pi jkt}$ [since the fundamental frequency $\omega_0 = 2\pi$ (why?)] determines the Fourier series a_k.

Consider the Nth approximation to its Fourier expansion, $x_N(t) = \sum_{k=-N}^{N} a_k \, e^{2\pi jkt}$:

a) By hand, plot $x(t)$ for more than one period, around $t = 0$, and evaluate the maximum, minimum and mean.

b) Either by calculating directly, or by using your class notes for a_k, simplify its Fourier series a_k using $T = 1$ and $\omega_0 = 2\pi$.

c) Using MATLAB, plot $x_N(t)$ for $N = 4$ over the same range as (a). Estimate the maximum, minimum and mean. The plot should resemble the plot in (a)—how good is it?

d) Repeat (c) for $N = 16$.

e) Repeat (c) for $N = 128$.

Optional Hint Because $x(t)$ is even, real, and has zero DC, the calculation may be simplified by using $a_k \, e^{2\pi jkt} + a_{-k} \, e^{-2\pi jkt} = a_k \left(e^{2\pi jkt} + e^{-2\pi jkt}\right) = 2a_k \cos(2\pi kt)$, giving $x_N(t) = \sum_{k=1}^{N} 2a_k \cos(2\pi kt)$ allowing you to avoid complex numbers in your program.

2) Consider the discrete periodic signal $x[n] = 1 + \sin(2\pi \frac{n}{8})$:

a) Plot $x[n]$ for the range [0,7].

b) Compute its Fourier series coefficients a_k for the ranges of k : [0,7] , [-3,4], and [8,15].

c) Use MATLAB to help solve this problem. Use the following commands:

```
n = [0:7];
x = 1+sin(2*pi*n/8);
A = fft(x)
```

How is `A` is related to a_k? There may be an overall constant factor disagreement. When comparing the elements of the array `A` to a_k, which value of k corresponds to the first element of `A` (i.e., `A(1)`)?

3) Consider the RLC circuit pictured in Figure P3.20 (page 254) in Oppenheim & Willsky, **except** that R, L, and C are unknown (i.e. keep them as "R", "L", and "C").
   a) Using an input signal of $V_0 \exp(j\omega t)$, and knowing that because the RLC circuit is an LTI system the output signal will be $H(j\omega)V_0 \exp(j\omega t)$, derive an expression for the transfer function $H(j\omega)$.
   b) For the values of R, L, and C in the book ($R = 1\Omega, L = 1\text{H}, C = 1\text{F}$), calculate $|H(j\omega)|$: the magnitude of $H(j\omega)$. Sketch it (by hand) for $0 \leq \omega < 3$ (i.e. Make sure the beginning and end values are right and get the general trend in between). Is this a low-pass, band-pass, or high-pass filter?

4) For each of the following signals $x(t)$, where a and L are unknown constants, sketch the signal and compute its Fourier Transform $X(j\omega)$:
   a) $x(t) = a\big(u(t) - u(t - L)\big)$
   b) $x(t) = a\big(u(t + L) - u(t)\big)$
   c) $x(t) = a\big(u(t + L/2) - u(t - L/2)\big)$
   d) $x(t) = au(t + L) + au(t + L/2) - au(t - L/2) - au(t - L)$

Note For the next problem, you may find this relation useful:

$$\int_{-\infty}^{\infty} \exp(jbx)\exp(-a^2x^2)\,dx = 2\int_{0}^{\infty} \cos(bx)\exp(-a^2x^2)\,dx = \frac{\sqrt{\pi}}{a}\exp(-b^2/4a^2)$$

5) Consider the Gaussian signal $x(t) = \exp(-t^2/2\sigma^2)$. [$\sigma^2$ is called the *variance* and parameterizes the width of the bell curve.]
   a) Sketch $x(t)$ for the range $[-2, +2]$ if $\sigma = 2$.
   b) Compute its Fourier Transform $X(j\omega)$.
   c) How does $|X(j\omega)|$ fall off as $|\omega| \to \infty$?
   d) Where is the maximum of $|X(j\omega)|$? How would this change if the signal was modified to be the related signal $x(t) = \exp(j\omega_0 t - t^2/2\sigma^2)$?