ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #7

This homework is worth 50% of other homeworks.

1) Consider the sawtooth function, $x(t) = \frac{t}{T}$ for $-\frac{T}{2} < t < \frac{T}{2}$, periodic with fundamental period T.

   a) Sketch $x(t)$ for more than one period, centered on $t = 0$.

   b) Sketch $\dot{x} = \frac{dx}{dt}$ for more than one period, centered on $t = 0$.

   c) For $\dot{x}$, compute its Fourier coefficients b_k, and its DC component b_0.

   d) From b_k, compute the Fourier coefficients of $x(t)$, a_k

   e) Note that this signal can be rewritten as $x(t) = \frac{t}{T} - \sum_{l=0}^{\infty} u\left(t - \frac{T}{2} - lT\right) + \sum_{l=1}^{\infty} u\left(-t + \frac{T}{2} - lT\right)$ for all t.

   Explain how this is so (feel free to illustrate your explanation in order to make it clear). Using this form, show that the derivative of this signal equals $\frac{1}{T} - \sum_{l=-\infty}^{\infty} \delta\left(t - \frac{T}{2} - lT\right)$. You'll likely need the fact that the impulse $\delta(t)$ is symmetric ($\delta(-t) = \delta(t)$) when simplifying it.

2) Consider the discrete periodic signal $x[n] = 1 + \sin(2\pi \frac{n}{8})$:

   a) Plot $x[n]$ for the range [0,7].

   b) Compute its Fourier series coefficients a_k for the ranges of k : [0,7] , [-3,4], and [8,15].

   c) Use MATLAB to help solve this problem. Use the following commands:

```
n = [0:7];
x = 1+sin(2*pi*n/8);
A = fft(x)
```

   How is `A` is related to a_k? There may be an overall constant factor disagreement. When comparing the elements of the array `A` to a_k, which value of k corresponds to the first element of `A` (i.e., `A(1)`)?

3) Consider the discrete periodic signal $x[n] = \sum_{l=-\infty}^{\infty} \delta[n - lN]$, the periodic impulse.

   a) Sketch $x[n]$ for more than one period, centered on $n = 0$. Use $N = 5$ *for the sketch only*.

   b) Compute its Fourier series coefficients a_k for all integers k.

   c) Repeat (b) for $x[n] = \sum_{l=-\infty}^{\infty} (\delta[n - lN] - \delta[n - 1 - lN])$, a periodic edge detector. Hint: use previous result.