

Homework #5

- 1) Consider the causal system defined by a circuit with input and output satisfying the differential equation $\dot{y}(t) + 2y(t) = 2x(t)$. For each of the different inputs below, find the output $y(t)$.
 - a) For $x(t) = u(t)$ (i.e. find the step response).
 - b) For $x(t) = \delta(t)$ (i.e. find the impulse response).
 - c) For $x(t) = e^{-t}u(t)$
 - d) For $x(t) = \cos(\omega t)u(t)$

- 2) For each different causal system find the impulse response $h[n]$:
 - a) $y[n] = (x[n] + x[n-1] + x[n-2]) / 3$
 - b) $y[n] = \alpha x[n] + \beta y[n-1]$, $|\beta| < 1$
 - c) $y[n] = x[n] + \beta y[n-2]$, $|\beta| < 1$
 - d) $y[n] = x[n] + x[n-1] + \beta y[n-2]$, $|\beta| < 1$

- 3) Consider the signal $x(t) = 2 \sin(\omega_0 t) + \cos(3\omega_0 t) + 5 \cos(4\omega_0 t + \theta)$:
 - a) What is the fundamental period?
 - b) Compute the Fourier coefficients a_k .

- 4) Fourier properties of signals based on periodic impulses
 - a) Consider the signal $x(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT)$
 - i) What is its fundamental period?
 - ii) Sketch $x(t)$ for at least one period, including a little on both sides of $t = 0$, using arrows to represent impulses (δ), as in *Oppenheim and Willsky* figure 3.12.
 - iii) Compute its Fourier coefficients a_k . You may need to evaluate its DC component a_0 separately.
 - iv) Is $x(t)$ even, odd, or neither?
 - v) Is a_k even, odd, or neither?
 - b) Repeat for $x(t) = \sum_{l=-\infty}^{\infty} \delta\left(t - \frac{T}{2} - lT\right)$. Hint: Start with (a), and use properties of the Fourier Series.

c) Repeat for $x(t) = \sum_{l=-\infty}^{\infty} \left(\delta(t-lT) - \delta\left(t - \frac{T}{2} - lT\right) \right)$. Hint: Start with (a) & (b).

d) Repeat for $x(t) = \int_{-\infty}^t \left(\sum_{l=-\infty}^{\infty} \left(\delta(t'-lT) - \delta\left(t' - \frac{T}{2} - lT\right) \right) \right) dt'$ Hint: Start with (c).