ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #4

Useful identities: $\cos(x) = (e^{jx} + e^{-jx})/2$

$\sin(x) = (e^{jx} - e^{-jx})/2j$

$f(|s|) = u(s)f(s) + u(-s)f(-s)$

For the problems below, we use the notation that y is the output of a system for an input x.

1) Consider the continuous LTI system with Impulse Response $h(t) = u(t)e^{-at}e^{j\omega_0 t}$, $a > 0$:
   a) For $x(t) = u(t)$, compute $y(t)$ and simplify.
   b) For $x(t) = u(t)e^{-bt}$, $b > 0$, $b \neq a$, compute $y(t)$ and simplify.

2) Consider the continuous LTI system with Impulse Response $h(t) = u(t)e^{-at}\cos(\omega_0 t)$, $a > 0$:
   a) For $x(t) = u(t)$, compute $y(t)$ and simplify.
   b) For $x(t) = u(t)e^{-bt}$, $b > 0$, $b \neq a$, compute $y(t)$ and simplify.

3) Consider the continuous LTI system with Impulse Response $h(s) = e^{-a|s|}$, $a > 0$:
   a) For $x(s) = e^{j\omega s}$, compute $y(s)$ and simplify.
   b) For $x(s) = \cos(\omega s)$, compute $y(s)$ and simplify.
   c) For $x(s) = \sin(\omega s)$, compute $y(s)$ and simplify.

4) Compute and simplify:
   a) $\int_{-\infty}^{\infty} \cos(\omega t)\delta(t - t_0)\,dt$
   b) $\int_{-\infty}^{\infty} \cos(\omega t)\delta'(t - t_0)\,dt$
   c) $\int_{-\infty}^{\infty} \cos(\omega t)\delta''(t - t_0)\,dt$
   d) $\cos(\omega t) * \delta(t)$
   e) $\cos(\omega t) * \delta(t - t_0)$
   f) $\cos(\omega t) * \delta'(t)$

5) Compute and simplify:
   a) $e^{j\omega t} * \left(e^{-bt}u(t)\right)$, $b > 0$

b) $\left(e^{-at}u(t)\right)*\left(e^{-bt}u(t)\right),\ a,b>0, a\neq b$

c) $e^{j\omega t}*\left(\left(e^{-at}u(t)\right)*\left(e^{-bt}u(t)\right)\right),\ a,b>0, a\neq b$

d) $\left(e^{j\omega t}*\left(e^{-at}u(t)\right)\right)*\left(e^{-bt}u(t)\right),\ a,b>0, a\neq b$

e) $\left(\left(e^{-bt}u(t)\right)*e^{j\omega t}\right)*\left(e^{-at}u(t)\right),\ a,b>0, a\neq b$

6) For each of the following LTI systems, compute the impulse response h, determine its inverse impulse response h_I, and demonstrate that $h * h_I = \delta$:

a) $y(s) = x(s - s_o)/2$

b) $y(t) = -\int_{-\infty}^{t} x(t')\,dt'$

c) $y[n] = \beta\,\mathrm{Diff}\{x[n]\} = \beta x[n] - \beta x[n-1], \beta \neq 0$

7) Consider the continuous LTI system with

$y[n] = \mathrm{Diff}\left\{\mathrm{Diff}\{x[n]\}\right\} = \mathrm{Diff}\left\{x[n] - x[n-1]\right\} = x[n] - 2x[n-1] + x[n-2]$:

a) Compute $h[n]$

b) Prove that its inverse impulse response is $h_I[n] = (n+1)u[n]$ by showing that $h_I[n]*h[n] = \delta[n]$ (you may want to use $(n+1)u[n] = u[n]*u[n]$)

8) Prove that each of the following are either stable or unstable systems:

a) $y(t) = -x(t-t_0) = \int_{-\infty}^{\infty} x(t')\left(-\delta\left((t-t')-t_0\right)\right)dt'$

b) $y(t) = \int_{-\infty}^{\infty} x(t')e^{-b(t-t')}u(t-t')\,dt', b>0$

c) $y(t) = \int_{-\infty}^{\infty} x(t')e^{-b(t-t')}\,dt', b>0$

d) $y(s) = \int_{-\infty}^{\infty} x(s')e^{-b(|s-s'|)}\,ds', b>0$

e) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\alpha^{-(n-k)},\ |\alpha|<1$

f) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\alpha^{-|n-k|},\ |\alpha|<1$