ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #3

All plots should be done by hand, not by computer (a calculator, if needed, is OK).

1) Consider the following systems, where $x \to [\text{System}] \to y$ *(these are the same as from last week)*

   a) $y(t) = \cosh\big(x(t)\big) = \frac{1}{2}\big(e^{x(t)} + e^{-x(t)}\big)$

   b) $y[n] = \text{Run}_{-\infty}\big(x[n]\big) = \sum_{n'=-\infty}^{n} x[n']$

   c) $y[n] = \big|\ x[n+1] - x[n]\ \big|$

   d) $\frac{d}{dt}y(t) + \omega y(t) = \omega^2 t x(t)$

   i) Which of the systems (a-d) are time invariant?

   ii) Which of the systems (a-d) are linear?

   iii) In a few words and/or equations, explain why system (b) is or isn't time invariant

   iv) In a few words and/or equations, explain why system (c) is or isn't time invariant

   v) In a few words and/or equations, explain why system (d) is or isn't time invariant

   vi) In a few words and/or equations, explain why system (a) is or isn't linear

   vii) In a few words and/or equations, explain why system (b) is or isn't linear

   viii) In a few words and/or equations, explain why system (c) is or isn't linear

2) For each of the following discrete LTI systems compute, and simplify, the output signal $y[n]$ when the input signal $x[n]$ is an unknown signal $f[n]$

   a) Impulse Response $h[n] = \delta[n - n_0]$; also, describe in a few words the effect of the system

   b) Impulse Response $h[n] = \delta[n] - \delta[n-1]$, the signed edge-detector

   c) Impulse Response $h[n] = -\delta[n+1] + 2\delta[n] - \delta[n-1]$, the signed curvature-detector

   d) Impulse Response $h[n] = (\delta[n+1] + 2\delta[n] + \delta[n-1])/4$, a symmetric blurring function

   e) Which of these four systems are causal, and why?

3) Let an input signal $x[n]$ be defined as $x[n] = \frac{\cos(\pi n/4)}{1+|n|}$. Using this input signal, you will plot the output signal for different systems.

a) First, plot the original signal $x[n]$ in the range [-6, +6] .

b) Now, using the discrete LTI system with Impulse Response $h[n] = \delta[n] - \delta[n-1]$ (the signed edge-detector system), plot the output $y[n]$ in the range [-6, +6]. It should be most extreme where $x[n]$ is changing fastest.

c) Similarly, using the discrete LTI system with Impulse Response $h[n] = -\delta[n+1] + 2\delta[n] - \delta[n-1]$ (the signed curvature-detector system), plot the new output $y[n]$ in the range [-6, +6]. It should be most extreme where $x[n]$ has the strongest curvature.

For the problems below, we use the notation that y is the output of a system for an input x.

4) Consider the discrete Linear Time Invariant (LTI) system with Impulse Response $h[n] = u[n]$:

a) For $x[n] = u[n]$, compute $y[n]$

b) For $x[n] = u[n]\alpha^n$, $|\alpha| < 1$, compute $y[n]$

5) Consider the following continuous LTI systems with different Impulse Responses, when given a common input signal $x(t) = u(t)$:

a) For $h(t) = u(t)$, compute $y(t)$ and simplify.

b) For $h(t) = u(t)\mathrm{e}^{-bt}$, $b > 0$, compute $y(t)$ and simplify.

c) For $h(t) = u(t)\mathrm{e}^{j\omega t}$, compute $y(t)$ and simplify.

d) For $h(t) = u(t)\cos(\omega t)$, compute $y(t)$ and simplify.

6) Consider the continuous LTI system with Impulse Response $h(t) = u(t)\mathrm{e}^{-at}$, $a > 0$:

a) For $x(t) = u(t)$, compute $y(t)$ and simplify.

b) For $x(t) = u(t)\mathrm{e}^{-at}$, compute $y(t)$ and simplify.

c) For $x(t) = u(t)\mathrm{e}^{j\omega t}$, compute $y(t)$ and simplify.

Useful identities:	$\cos(x) = (\mathrm{e}^{jx} + \mathrm{e}^{-jx})/2$
	$\sin(x) = (\mathrm{e}^{jx} - \mathrm{e}^{-jx})/2j$