

**Homework #2**

**All plots should be done by hand, not by computer (a calculator, if needed, is OK).**

1) Consider the continuous signals  $x(s) = \max(\sin(2\pi s), 0)$  and  $y(s) = \min(\sin(2\pi s), 0)$

$$\text{where } \max(a, b) = \begin{cases} a & a \geq b \\ b & a < b \end{cases} \text{ and } \min(a, b) = \begin{cases} b & a \geq b \\ a & a < b \end{cases} .$$

- a) Is  $x(s)$  periodic? If so, compute the fundamental period.
- b) Is  $y(s)$  periodic? If so, compute the fundamental period.
- c) Consider the signal  $(x(s) - y(s))$ . Is it periodic? If so, compute the fundamental period, and reconcile your answer with the answers to (a) and (b).

2) Consider the discrete signals  $x[n] = \begin{cases} 0 & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$  and  $y[n] = \sin(n)$ .

- a) Is  $x[n]$  periodic? If so, compute the fundamental period.
- b) Is  $y[n]$  periodic? If so, compute the fundamental period.

3) For each of the following continuous signals  $x(t)$ , compute,  $\text{Ev}\{x(t)\}$  i.e. the even part of  $x(t)$ , and  $\text{Od}\{x(t)\}$ , i.e. the odd part of  $x(t)$ .

a)  $x(t) = C e^{j\omega_o t}$ , where  $C = e^{j\frac{\pi}{2}}$ .

b)  $x(t) = \frac{1}{1 + 2t}$ .

c)  $x(t) = 2$ .

4) Consider the continuous signal  $x(t) = C e^{j\omega_o t}$

a) For  $C = \frac{1 + j}{\sqrt{2}} = e^{+j\frac{\pi}{4}}$ ,  $\omega_o = \frac{4}{3}\pi$ , on separate graphs, plot  $\text{Re}(x(t))$  and  $\text{Im}(x(t))$  in the range  $[-3, +3]$ ,

and compute the fundamental period of  $x(t)$ .

b) For  $C = 1.5 \text{ Volts}$ ,  $\omega_o = 2\pi f_0$ ,  $f_0 = 2 \text{ kHz}$ , on separate graphs, plot  $\text{Re}(x(t))$  and  $\text{Im}(x(t))$  in the range  $[-1 \text{ ms}, +1 \text{ ms}]$ , and compute the fundamental period of  $x(t)$  (with the correct units).

5) Consider the continuous signal  $x(t) = C e^{\alpha t}$ , where  $C = \sqrt{2}(1 + j) = 2e^{+j\frac{\pi}{4}}$ ,  $\alpha = 1 + 2\pi j$ .

a) Plot  $|x(t)|$  and  $-|x(t)|$  and  $\text{Re}(x(t))$ , on the same graph, in the range  $[-2, +2]$ .

- b) Plot  $|x(t)|$  and  $-|x(t)|$  and  $\text{Im}(x(t))$ , on the same graph, in the range  $[-2, +2]$  .
- 6) Consider the discrete signal:  $x[n] = C\alpha^n$  .
- Plot  $x[n]$  in the range  $[-2, +2]$ , for  $C = 3, \alpha = 2$  .
  - Plot  $x[n]$  in the range  $[-2, +2]$ , for  $C = 3, \alpha = 2^{-1}$  .
  - Plot  $x[n]$  in the range  $[-2, +2]$ , for  $C = 3, \alpha = -2$  .
  - Plot  $x[n]$  in the range  $[-2, +2]$ , for  $C = -3, \alpha = 2$  .
- 7) Consider a discrete signal  $x[n] = Ce^{j\omega_0 n}$  which is periodic with  $N = 6$  .
- List the allowed values of  $\omega_0$  in the range  $[0, 2\pi]$  .
  - List the allowed values of  $\omega_0$  in the range  $[-\pi, +\pi]$  .
- 8) Consider the following systems, where  $x \rightarrow [\text{System}] \rightarrow y$
- $y(t) = \cosh(x(t)) = \frac{1}{2}(e^{x(t)} + e^{-x(t)})$
  - $y[n] = \text{Run}_{-\infty}(x[n]) = \sum_{n'=-\infty}^n x[n']$
  - $y[n] = |x[n+1] - x[n]|$
  - $\frac{d}{dt}y(t) + \omega y(t) = \omega^2 tx(t)$
- Which of the systems (a-c) are instantaneous/ultralocal/memoryless?
  - Which of the systems (a-d) are invertible?
  - Which of the systems (a-c) are causal?
  - In a few words and/or equations, explain why system (a) is or isn't instantaneous/ultralocal/memoryless
  - In a few words and/or equations, explain why system (b) is or isn't instantaneous/ultralocal/memoryless
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  - ~~In a few words and/or equations, explain why system (b) is or isn't instantaneous/ultralocal/memoryless~~
  - In a few words and/or equations, explain why system (a) is or isn't invertible
  - In a few words and/or equations, explain why system (b) is or isn't invertible
  - In a few words and/or equations, explain why system (c) is or isn't invertible
  - In a few words and/or equations, explain why system (d) is or isn't invertible
  - In a few words and/or equations, explain why system (b) is or isn't causal
  - In a few words and/or equations, explain why system (c) is or isn't causal