ENEE 322

Sections 0201 & 0202 Spring 2018

Homework #1

All plots should be done by hand, not by computer (a calculator, if needed, is OK).

1) Consider the continuous signal: $x(t) = C_0 \sin\left(2\pi f_0 t\right)$

   a) Compute the power of $x(t)$

   b) Compute the energy in $x(t)$ for the single cycle between $t_1 = 0$ and $t_2 = 1/f_0$

   c) for the single cycle between $t_1 = -1/(2f_0)$ and $t_2 = 1/(2f_0)$

   d) for all cycles, between $t_1 = -\infty$ and $t_2 = +\infty$

   e) Compute the average power in $x(t)$ for the single cycle between $t_1 = 0$ and $t_2 = 1/f_0$

   f) for the single cycle between $t_1 = -1/(2f_0)$ and $t_2 = 1/(2f_0)$

   g) for all cycles between $t_1 = -\infty$ and $t_2 = +\infty$

2) Consider the discrete signal: $x[n] = B_0 \left(\frac{1}{3}\right)^{n/2}$ (and see Note after the last problem)

   a) Compute the power of $x[n]$

   b) Compute the energy in $x[n]$ from $n_1 = 1$ to $n_2 = 2$

   c) from $n_1 = 1$ to $n_2 = 3$

   d) from $n_1 = 1$ to $n_2 = \infty$ (you may want to see **note** after the last problem)

   e) Compute the average power in $x[n]$ from $n_1 = 1$ to $n_2 = 2$

   f) from $n_1 = 1$ to $n_2 = 3$

   g) from $n_1 = 1$ to $n_2 = \infty$

3) Consider the discrete signal $x[n] = \sin\left(n\pi/4\right)$

   a) Plot $x[n] \to x[-n]$ in the range [–5,+5]

   b) Plot $x[n] \to x[n-2]$ in the range [–5,+5]

   c) Plot $\text{Diff}(x[n]) := x[n] - x[n-1]$ in the range [–5,+5]

   d) Plot $\text{Run}_{-5}(x[n]) := \sum_{n'=-5}^{n} x[n']$ in the range [–5,+5]

4) Consider the continuous signal $x(s) = \max\left(\sin(2\pi s), 0\right)$, where $\max\left(a,b\right) = \begin{cases} a & a \ge b \\ b & a < b \end{cases}$

   a) Plot $x(s)$ in the range [–1,+1] (i.e., the "half-wave rectification" of $\sin(2\pi s)$).

   b) Plot $x(s) \to x(-s)$ in the range [–1,+1]

   c) Plot $x(s) \to x(s+\frac{1}{2})$ in the range [–1,+1]

d) Plot $x(s) \to x\left(\frac{1}{2}(s+\frac{1}{2})\right)$ in the range [–1,+1]

e) Compute the average power in $x(s)$ between $s_1 = -\frac{1}{2}$ and $s_2 = +\frac{1}{2}$, **and** between $s_1 = -\infty$ and $s_2 = +\infty$

5) Consider the discrete impulse and step functions, $\delta[n]$ and $u[n]$

a) Compute the following three values: $\delta[0]$ $\delta[1]$ $\delta[-2]$

b) Compute the following three values: $u[0]$ $u[1]$ $u[-2]$

6) Consider these signals

a) Plot $x[n] = \delta[n]$ in the range [–5,+5]

b) Plot $x[n] = 1$ in the range [–5,+5]

c) Plot $x[n] = u[n]$ in the range [–5,+5]

d) Plot $x[n] = u[n-3]$ in the range [–5,+5]

e) Plot $x[n] = u[n]\text{-}u[n\text{-}3]$ in the range [–5,+5]

f) Plot $x[n] = \delta[n]+\delta[n\text{-}1]+\delta[n\text{-}2]$ in the range [–5,+5]

7) Consider the discrete signal $x[n] = \sin\left(n\pi/8\right)$

a) Compute $\sum_{n=-\infty}^{+\infty} x[n]\delta[n+2]$.

b) Compute $\sum_{n=-\infty}^{+\infty} x[n]\delta[n-n_0]$.

c) Compute $\sum_{n=-\infty}^{+\infty} \left(2n^2 - 4n + 6\right)\delta[n-1]$.

d) Compute $\sum_{n=-\infty}^{+\infty} \left(2n^2 - 4n + 6\right)\delta[n-n_0]$.

e) Compute $\sum_{n=-\infty}^{+\infty} y[n]\delta[n-n_0]$ where $y[n]$ is unknown.

8) Consider the continuous signal $x(t) = \exp\left(j2\pi ft\right)$

a) Compute $\int_{-\infty}^{+\infty} x(t)\delta(t+\frac{1}{2f})\,dt$.

b) Compute $\int_{-\infty}^{+\infty} x(t)\delta(t-1/f)\,dt$.

c) Compute $\int_{-\infty}^{+\infty} t\,e^{-t}\,\delta(t+2)\,dt$.

d) Compute $\int_{-\infty}^{+\infty} y(t)\delta(t-t_0)\,dt$ where $y(t)$ is unknown.

Note: Really easy ways to total a geometric sum (so easy you could use them even without this sheet)

Let $\boxed{S_0 = \sum_{m=0}^{M} \alpha^m} = 1 + \alpha + \alpha^2 + \ldots + \alpha^M$ be the sum in question. Then $\alpha S_0 = \alpha + \alpha^2 + \ldots + \alpha^M + \alpha^{M+1}$, and so $S_0 - \alpha S_0 = (1 + \alpha + \alpha^2 + \ldots + \alpha^M) - (\alpha + \alpha^2 + \ldots + \alpha^M + \alpha^{M+1}) = 1 - \alpha^{M+1}$. Therefore $\boxed{S_0 = (1 - \alpha^{M+1}) / (1 - \alpha)}$.

It also works for $\boxed{S_1 = \sum_{m=1}^{M} \alpha^m} = \alpha + \alpha^2 + \ldots + \alpha^M$: $S_1 - \alpha S_1 = (\alpha + \alpha^2 + \ldots + \alpha^M) - (\alpha^2 + \ldots + \alpha^M + \alpha^{M+1}) = \alpha - \alpha^{M+1}$, from which we see $\boxed{S_1 = (\alpha - \alpha^{M+1}) / (1 - \alpha)}$.

When $M \to \infty$, the sums simplify even more, as long as $\alpha^\infty \to 0$:

For $\boxed{S_0 = \sum_{m=0}^{\infty} \alpha^m}$, $S_0 - \alpha S_0 = (1 + \alpha + \alpha^2 + \ldots) - (\alpha + \alpha^2 + \ldots) = 1$, so $\boxed{S_0 = 1 / (1 - \alpha)}$.

For $\boxed{S_1 = \sum_{m=1}^{\infty} \alpha^m}$, $S_1 - \alpha S_1 = (\alpha + \alpha^2 + \ldots) - (\alpha^2 + \ldots) = \alpha$, so $\boxed{S_1 = \alpha / (1 - \alpha)}$.