ENEE 222 0201/2 HOMEWORK ASSIGNMENT 11 Due Tue 05/13/14

Problem 11A

Consider the FIR filter with input-output relationship

$$y[n] \;=\; x[n] + 3x[n-1] + 3x[n-3] + x[n-4]\;, \qquad n \in \mathbf{Z}$$

(Note the missing, i.e., zero, coefficient.)

(i) Determine the response $y^{(i)}[\,\cdot\,]$ of the filter to each of the input signals given by the equations below (valid for all $n \in \mathbf{Z}$).

$$\begin{aligned}
x^{(1)}[n] &= (3/4)^n && \textbf{(2 pts.)}\\
x^{(2)}[n] &= (-4/3)^n && \textbf{(2 pts.)}\\
x^{(3)}[n] &= 1 + 3^{-n} && \textbf{(2 pts.)}\\
x^{(4)}[n] &= \cos(n(\pi/6) + 2.5) && \textbf{(3 pts.)}\\
x^{(5)}[n] &= 2^{-n}\cdot \cos(n\pi/6) && \textbf{(4 pts.)}
\end{aligned}$$

(ii) (5 pts.) The filter above is connected in series (cascade) with a filter having input-output relationship

$$y[n] \;=\; x[n] - 2x[n-1] + x[n-2]\;, \qquad n \in \mathbf{Z}$$

Determine the system function $H(z)$ of the two-filter cascade.

(iii) (2 pts.) Write out the input-output relationship of the two-filter cascade.

Problem 11B

Consider the FIR filter with coefficient vector $\mathbf{b} = \begin{bmatrix} 1 & 3 & 0 & -3 & -1 \end{bmatrix}^T$.

The following MATLAB script computes a segment of the filter output sequence $y^{(1)}[\,\cdot\,]$ for a periodic input sequence $x^{(1)}[\,\cdot\,]$ of period $L = 6$. Specifically, the vector `y1` below equals $y^{(1)}[-1:4]$.

```
b  = [1 3 0 -3 -1].' ;
H  = fft(b,6) ;
x1 = [1 2 4 -1 -2 -4].' ;
X1 = fft(x1) ;
Y1 = H.*X1 ;
y1 = ifft(Y1)
```

(i) (3 pts.) Display the vector $x^{(1)}[0:5]$. (*It is not the same as* `x1`, *introduced above.*)

(ii) (5 pts.) Compute $y^{(1)}[-1:4]$ by hand using a circular convolution, or, equivalently, using the filter's input-output relationship.

For parts (iii) and (iv) below, consider the following MATLAB script, which computes a segment of the filter output sequence $y^{(2)}[\,\cdot\,]$ for a periodic input sequence $x^{(2)}[\,\cdot\,]$ of period $L = 3$. Specifically, the vector `y2` equals $y^{(2)}[-1:4]$.

```
b  = [1 3 0 -3 -1].' ;
H  = fft(b,6) ;
x2 = [2 -1 5 2 -1 5].' ;
X2 = fft(x2) ;
Y2 = H.*X2 ;
y2 = ifft(Y2)
```

(iii) (2 pts.) Display the vector $x^{(2)}[0:2]$.

(iv) (5 pts.) Compute $y^{(2)}[-1:4]$ by hand using a circular convolution, based on the filter's input-output relationship.

(v) (5 pts.) Fully explain the relationship between `y3` computed below and `y2` computed earlier.

```
b  = [1 3 0 -3 -1].' ;
H  = fft(b,6) ;
H  = H(1:2:6) ;
x3 = [2 -1 5].' ;
X3 = fft(x3) ;
Y3 = H.*X3 ;
y3 = ifft(Y3)
```

Problem 11C

Consider the FIR filter with impulse response given by

$$h[n] \;=\; b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + b_3\delta[n-3] \;+ b_4\delta[n-4] + b_5\delta[n-5]$$

It is known that the response of the filter to the input sequence

$$x[0:3] \;=\; \begin{bmatrix} 1 & -2 & 3 & -4 \end{bmatrix}^T ; \qquad x[n] = 0 \text{ for } n < 0 \text{ and } n > 3$$

is the output sequence given by

$$y[0:8] \;=\; \begin{bmatrix} 1 & -3 & 8 & -16 & 20 & -24 & 17 & -7 & 4 \end{bmatrix}^T$$

and $y[n] = 0$ for $n < 0$ and $n > 8$.

(i) (4 pts.) Use the `FFT` command in MATLAB to determine the filter coefficient vector $\mathbf{b}$. (Alternatively, $\mathbf{b}$ can be determined by solving a triangular system of linear equations.)

(ii) (2 pts.) Using linearity and time invariance, write an equation for the response of the filter to the input sequence given by

$$x^{(1)}[n] \;=\; \delta[n+3] - \delta[n-3]$$

(iii) (3 pts.) Using the system function $H(z)$ or otherwise, determine the response of the filter to the exponential input

$$x^{(2)}[n] \;=\; (-2)^{-n} , \qquad n \in \mathbf{Z}$$

(iv) (7 pts.) Using convolution, determine the response of the filter to the finite-duration sequence

$$x^{(3)}[n] \;=\; 16\delta[n] - 8\delta[n-1] + 4\delta[n-2] - 2\delta[n-3] + \delta[n-4]$$

(Make sure to specify the output sequence completely.)

(v) (4 pts.) Explain how some of the values computed in (iv) above can be combined with the answer to part (iii) to yield the response of the filter to the one-sided exponential input

$$x^{(4)}[n] = \begin{cases} (-2)^{-n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Determine that response.

Solved Examples

S **11.1** (*P* **4.4** in textbook). Consider the FIR filter whose input $\mathbf{x}$ and output $\mathbf{y}$ are related by

$$y[n] = x[n] - x[n-1] - x[n-2] + x[n-3]$$

(i) Write out an expression for the system function $H(z)$.

(ii) Express $|H(e^{j\omega})|^2$ in terms of cosines only. Plot $|H(e^{j\omega})|$ as a function of ω.

(iii) Determine the output $y[n]$ when the input sequence $\mathbf{x}$ is given by each of the following expressions (where $n \in \mathbf{Z}$):

- $x[n] = 1$
- $x[n] = (-1)^n$
- $x[n] = e^{j\pi n/4}$
- $x[n] = \cos(\pi n/4 + \phi)$
- $x[n] = 2^{-n}$
- $x[n] = 2^{-n}\cos(\pi n/4)$

(In all cases except the third, your answer should involve real-valued terms only.)

S **11.2** (*P* **4.8** in textbook). Consider two FIR filters with coefficient vectors $\mathbf{b}$ and $\mathbf{c}$, where

$$\mathbf{b} = \begin{bmatrix} 3 & 2 & 1 & 2 & 3 \end{bmatrix}^T$$

and

$$\mathbf{c} = \begin{bmatrix} 1 & -2 & 2 & -1 \end{bmatrix}^T$$

(i) Determine the system function $H(z)$ of the cascade. Is the cascade also a FIR filter? If so, determine its coefficient vector.

(ii) Express the amplitude response of the cascade as a sum of sines or cosines (as appropriate) with real-valued coefficients.

S **11.3** (*P* **4.9** in textbook). Consider the FIR filter with coefficient vector

$$\mathbf{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$$

Two copies of this filter are connected in series (cascade).

(i) Determine the system function $H(z)$ of the cascade. Is the cascade also a FIR filter? If so, determine its coefficient vector.

(ii) Determine the response $y[n]$ of the cascade to the sinusoidal input sequence

$$x[n] = \cos\left(\frac{n\pi}{2}\right), \qquad n \in \mathbf{Z}$$

S **11.4** (*P* **4.3** in textbook). Consider the signal sequence $\mathbf{x}$ defined by

$$x[n] = \cos\left(\frac{3\pi n}{14} - 1.8\right) + 2\cos\left(\frac{18\pi n}{35} - 0.7\right) + 6\cos\left(\frac{17\pi n}{24} + 2.0\right)$$

(i) Is the sequence periodic, and if so, what is its period?

(ii) Sketch the amplitude and phase spectra of $\mathbf{x}$ (both of which are line spectra).

S **11.5** (*P* **4.7** in textbook). Consider a FIR filter whose input $\mathbf{x}$ and output $\mathbf{y}$ are related by

$$y[n] = \sum_{k=0}^{8} b_k x[n-k] ,$$

where the coefficient vector $\mathbf{b}$ is given by

$$\mathbf{b} = [\ 1 \quad 2 \quad -2 \quad -1 \quad 4 \quad -1 \quad -2 \quad 2 \quad 1\ ]^T$$

Let the input $\mathbf{x}$ be an infinite periodic sequence with period $L = 7$, and such that

$$x[0:6] = [\ 1 \quad -1 \quad 0 \quad 3 \quad 1 \quad -2 \quad 0\ ]^T$$

Using DFT's (and MATLAB), determine the first period $y[0:6]$ of the output sequence $\mathbf{y}$.

S **11.6** (*P* **4.13** in textbook). Consider the FIR filter with impulse response $\mathbf{h}$ given by

$$h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-3] - 2\delta[n-4]$$

(i) Without using convolution, determine the response $\mathbf{y}$ of the filter to the input signal $\mathbf{x}$ given by

$$x[n] = \delta[n+1] - \delta[n-1]$$

(*Hint:* Express $\mathbf{y}$ in terms of $\mathbf{h}$.)

(ii) Now let the input signal $\mathbf{x}$ be given by

$$x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$

Using convolution, determine the output signal $\mathbf{y}$.

(iii) Obtain the answer to **(ii)** by direct multiplication of the z-transforms $H(z)$ and $X(z)$.

S **11.7** (*P* **4.14** in textbook). Consider the signal sequence

$$x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine the convolution

$$\mathbf{y} = \mathbf{x} * \mathbf{x}$$

For what values of n is $y[n]$ nonzero?

(ii) By how many instants should $\mathbf{y}$ be delayed (or advanced) in time so that the resulting signal is symmetric about $n = 0$? What is the DTFT of that signal?

S **11.8** (*P* **4.15** in textbook). Define $\mathbf{x}$ by

$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

and let $\mathbf{h}$ be the (*infinite* impulse response) sequence given by

$$h[n] = \begin{cases} \alpha^n, & n \geq 0; \\ 0, & n < 0. \end{cases}$$

Compute $\mathbf{y} = \mathbf{x} * \mathbf{h}$. Sketch your answer in the case $\alpha = 2$.