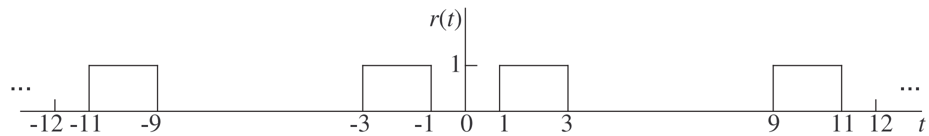
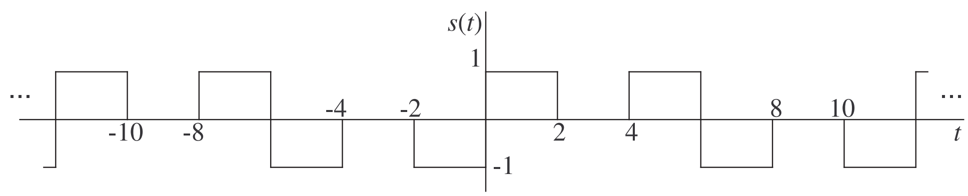
ENEE 222 0201/2 HOMEWORK ASSIGNMENT 10 Due Tue 05/06/14

Problem 10A

(i) (3 pts.) Express the signal $r(t)$ of period $T_0 = 12$ (seconds) shown below as the sum of two symmetric rectangular pulse trains. Hence derive the Fourier series coefficients $\{R_k\}$ of $r(t)$.

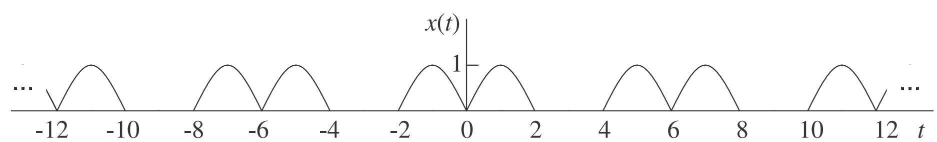


(ii) (6 pts.) Using the time delay property, derive the Fourier series coefficients $\{S_k\}$ of $s(t)$ shown below. Verify that S_k is purely imaginary and such that $S_{-k} = -S_k$. Also verify that $S_k = 0$ for all even indices k.



(iii) (4 pts.) Use MATLAB to graph the output of an ideal lowpass filter with cutoff frequency 4.1 Hz when the input is $s(t)$. Display two periods corresponding to the time interval $[0, 24)$ with 256 samples per period. Submit commands and graphs.

(iv) (7 pts.) Using the result of (ii) and the modulation property, derive the Fourier series coefficients $\{X_k\}$ of the periodic signal $x(t)$ shown below (curved segments are sinusoidal). Use the same fundamental period as for $s(t)$, namely $T_0 = 12$. Simplify your answer as much as possible. Verify that X_k is real, that $X_{-k} = X_k$, and that $X_k = 0$ for all odd indices k. By inspection of $x(t)$, why would you expect all of these properties to hold?



Problem 10B

Consider the FIR filter given by the input-output relationship

$$y[n] = -\frac{1}{12}x[n] + \frac{2}{3}x[n-1] - \frac{2}{3}x[n-3] + \frac{1}{12}x[n-4]\ , \qquad n \in \mathbf{Z}$$

(Note that the coefficient of $x[n-2]$ equals zero.)

(i) (4 pts.) Show that the input sequences defined for all n by $x^{(1)}[n] = 1$ and $x^{(2)}[n] = (-1)^n$ both result in output sequences which are identically equal to zero.

(ii) (4 pts.) Write MATLAB code which computes and plots the magnitude and phase response of the filter at 1024 equally spaced frequencies in $[0, 2\pi)$.

(iii) (6 pts.) Express the filter's frequency response in the form

$$H(e^{j\omega}) = je^{-j(\omega M/2)}F(\omega)$$

where $F(\omega)$ is a real-valued sum of sines.

This FIR filter can be used for numerical differentiation. Specifically, if the input sequence $x[\cdot]$ consists of dense samples of a smooth function $x(\cdot)$, i.e., $x[n] = x(n\Delta)$ for Δ small, it can be shown that the output sequence $y[\cdot]$ is approximately proportional to the derivative $x'(\cdot)$ with a two-sample delay:

$$y[n] \simeq \Delta \cdot x'((n-2)\Delta) ,$$

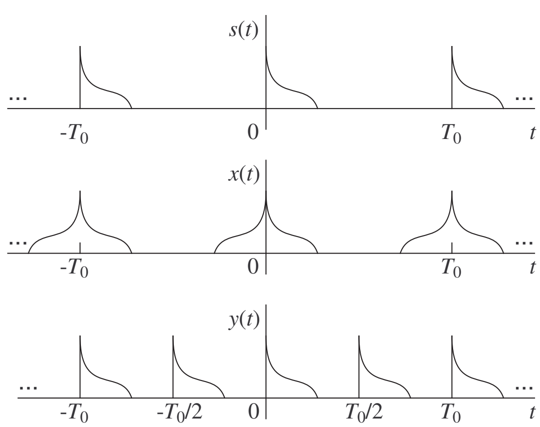
the approximation being exact for cubic (or lower-degree) polynomials. This means that the input sequence $x[n] = e^{j\omega n}$ obtained from the continuous-time sinusoid $x(t) = e^{j\Omega t}$ should, for small values of $\omega = \Omega\Delta$, result in an output approximately equal to $y[n] = j\omega e^{j\omega n}$. Thus the magnitude response is approximately equal to $|j\omega| = \omega$ for small (and positive) ω.

(iv) (4 pts.) Show that if the Taylor series-based approximation $\sin\theta \simeq \theta - \theta^3/6$ (for small θ) is used in the expression for $F(\omega)$ obtained in part **(iii)**, then $H(e^{j\omega})$ is approximately equal to $j\omega$ for small values of ω.

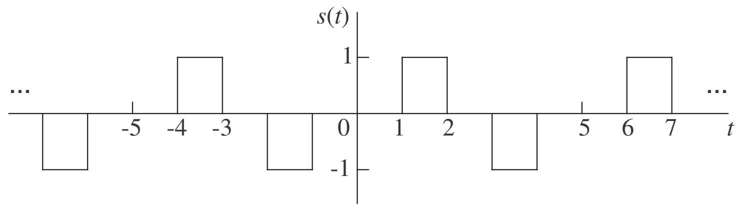
(v) (2 pts.) Use the `HOLD` feature to overlay a straight line of slope 1 on the left half of the magnitude response plot generated in part **(ii)**. This should confirm that $|H(e^{j\omega})| \simeq \omega$ for small positive ω. Submit both magnitude and phase response plots.

Solved Examples

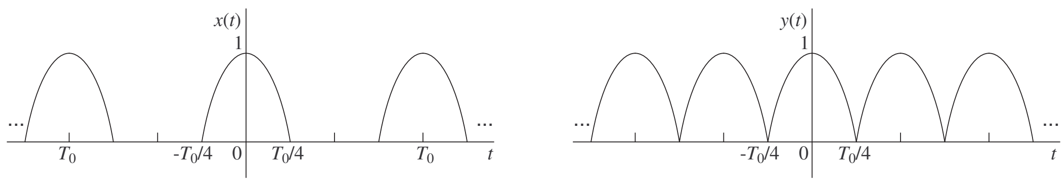
S **10.1.** Express the Fourier series coefficients of $x(t)$ and $y(t)$ below in terms of those of $s(t)$.



S **10.2.** Determine the Fourier series coefficients of the periodic signal $s(t)$ of period $T_0 = 5$ shown below.



S **10.3.** Use two different methods to express the complex Fourier series coefficients of $y(t)$ (full-wave rectified sinusoid) in terms of those of $x(t)$ (half-wave rectified sinusoid), as shown below.



S **10.4** (*P* **4.5** in textbook). Consider the FIR filter

$$y[n] = x[n] - 3x[n-1] + x[n-2] + x[n-3] - 3x[n-4] + x[n-5]$$

(i) Write MATLAB code which includes the function `fft`, and which computes the magnitude and phase response of the filter at 256 equally spaced frequencies between 0 and $2\pi(1 - 256^{-1})$.

(ii) Express the frequency response of the filter in the form

$$e^{-j\alpha\omega}F(\omega)$$

where $F(\omega)$ is a real-valued sum of cosines.

(iii) Determine the response $y[n]$ of the filter to the exponential input sequence

$$x[n] = \left(\frac{1}{2}\right)^n, \qquad n \in \mathbf{Z}$$

S **10.5** (*P* **4.6** in textbook). The MATLAB code

```
a = [ 1 -3 5 -3 1 ].' ;
H = fft(a,500);
A = abs(H);
q = angle(H);
```

computes the magnitude response `A` and phase response `q` of a FIR filter over 500 equally spaced frequencies in the interval $[0, 2\pi)$.

(i) If $\mathbf{x}$ and $\mathbf{y}$ are (respectively) the input and output sequences of that filter, write an expression for $y[n]$ in terms of values of $\mathbf{x}$.

(ii) Determine the output $\mathbf{y}$ of the filter when the input $\mathbf{x}$ is given by

$$x[n] = \left(\frac{1}{3}\right)^n, \qquad n \in \mathbf{Z}$$

(iii) Express the frequency response of the filter in the form

$$e^{-j\alpha\omega}F(\omega)$$

where $F(\omega)$ is a real-valued sum of cosines.