ENEE 222 0201/2 HOMEWORK ASSIGNMENT 9 Due Tue 04/29/14

Problem 9A

Let

$$s(t) \;=\; 2.3 \,+\, 7.8\cos(90\pi t + 1.2) \,+\, 5.4\cos(270\pi t - 2.5) \,+\, 2.6\cos(315\pi t + 0.3)\ ,$$

where t is in seconds.

(i) (4 pts.) Is $s(t)$ periodic? If so, what is its fundamental period T_0 and angular frequency Ω_0?

(ii) (6 pts.) If

$$s(t) \;=\; \sum_{k=-\infty}^{\infty} S_k e^{jk\Omega_0 t}\ ,$$

determine the value of each coefficient S_k (it suffices to leave it in polar form).

(iii) (4 pts.) Suppose that $s(t)$ is sampled every $T_s = T_0/N$ seconds, where N is an integer, to produce

$$s[n] \;=\; s(nT_s)$$

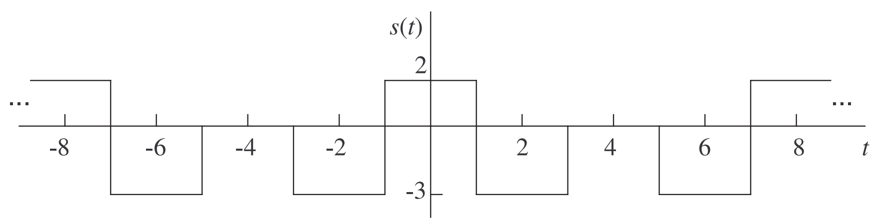
Write an equation for $s[n]$ in terms of real sinusoids. What are the frequencies of these sinusoids? Are they Fourier frequencies for an N-point vector?

(iv) (6 pts.) Let $N = 256$. Use the `IFFT` function in MATLAB to generate the vector $s[0:255]$, which consists of N uniform samples of $s(t)$ over its first period $[0, T_0)$. *Submit the commands used and the resulting plot; do not include a printout of the vector.*

Problem 9B

(i) (2 pts.) In the lecture notes, you will find the Fourier series for the symmetric (even) rectangular pulse train of unit height and duty factor α. Write down *both* the complex and real (cosines-only) form of the series for a fundamental period $T_0 = 8$.

(ii) (5 pts.) Express the periodic signal $s(t)$ (of period $T_0 = 8$) shown below as a sum of two symmetric rectangular pulse trains of the same period. Using the result of (i), derive the complex Fourier series coefficients $\{S_k\}$. Also, write down the real (cosines-only) form of the series for $s(t)$.

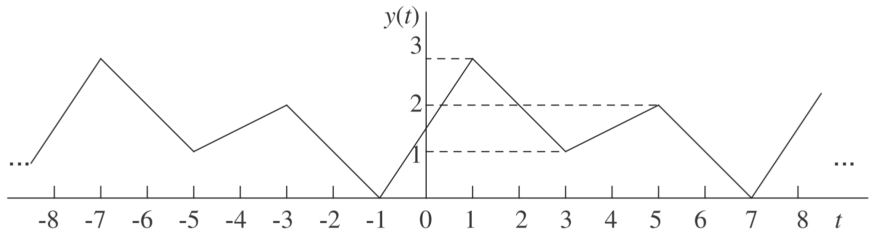


(iii) (3 pts.) Sketch the periodic signal $x(t)$ which has period $T_0 = 8$ (i.e., same as $s(t)$) and complex Fourier series coefficients given by

$$X_k \;=\; \begin{cases} S_k, & k \neq 0; \\ 0, & k = 0. \end{cases}$$

For (iv)–(vi), consider the periodic signal $y(t)$ shown below.

(iv) (2 pt.) Determine the mean value (or DC offset) of $y(t)$.



(v) (3 pts.) Determine the values taken by the derivative $dy(t)/dt$ over one period, e.g., for $t \in (-1, 7]$. What is the relationship between $dy(t)/dt$ and the signal $x(t)$ of part (iii)? (*It is a simple relationship.*)

(vi) (5 pts.) The real form of the Fourier series expansion of $y(t)$ is

$$y(t) \;=\; Y_0 \;+\; 2\sum_{k=1}^{\infty} B_k \sin(k\Omega_0 t)$$

Using the results of (ii), (iv) and (v), together with the well-known identity $(d/dt)\sin(at) = a\cos(at)$, determine Y_0 and B_k for $k \geq 1$.

(*Note:* The signal $y(t) - Y_0$ is antisymmetric, or, "odd", about $t = 0$. The *complex* Fourier series coefficients Y_k are related to B_k by $Y_k = jB_{-k}$ for $k < 0$; and $Y_k = -jB_k$ for $k > 0$.)

Solved Examples

S **9.1** Determine the fundamental period T_0 of

$$s(t) \;=\; 8\cos(46\pi t)\cos(50\pi t)\cos(60\pi t)$$

S **9.2** If

$$s(t) \;=\; 11 + \cos(30\pi t) - 5\sin(37.5\pi t) - 9\cos(52.5\pi t) + 2\sin(52.5\pi t) \;,$$

determine the fundamental period T_0 of $s(t)$ and its complex Fourier series coefficients in Cartesian form. Is $s(t)$ symmetric (even) or antisymmetric (odd) about $t = 0$?

S **9.3** The real-valued signal $s(t)$ has period $T_0 = 0.01$ seconds and is bandlimited to 850 Hz. As a result, its complex Fourier series is a sum of seventeen terms. The file `data10S.txt` contains seventeen uniform samples of $s(t)$, taken every $T_0/17$ seconds starting at $t = 0$. Using the `FFT` and `IFFT` functions, determine the Fourier series coefficients $\{S_k\}$ and plot 340 uniform samples of the signal over $[0, T_0)$.

S **9.4** Derive a real-valued closed-form expression (i.e., no sum over k) for the periodic signal $s(t)$ of fundamental period T_0, whose complex Fourier series coefficients are given by

$$S_k \;=\; \begin{cases} 1, & |k| \leq M; \\ 0, & \text{otherwise.} \end{cases}$$

Where have you encountered this function before? (*Hint*: Use a finite geometric sum.)

S **9.5** Derive a real-valued closed-form expression (i.e., no sum over k) for the periodic signal $s(t)$ of fundamental period T_0, whose complex Fourier series coefficients are given by

$$S_k \;=\; 2^{-|k|}$$

for all $k \in \mathbf{Z}$. Also, sketch the signal. (*Hint*: Use two infinite geometric sums.)