ENEE 222 0201/2 HOMEWORK ASSIGNMENT 8 Due Tue 04/15/14

Problem 8A

(i) (6 pts.) Determine the circular convolution $\mathbf{s} = \mathbf{x} \circledast \mathbf{y}$ of

$$\mathbf{x} = \begin{bmatrix} 4 & -2 & 1 & -4 \end{bmatrix}^T$$

and

$$\mathbf{y} = \begin{bmatrix} 1 & 3 & -3 & -1 \end{bmatrix}^T$$

Using the `FFT` function in MATLAB, verify that the DFTs $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{S}$ satisfy $\mathbf{X} \diamond \mathbf{Y} = \mathbf{S}$.

(ii) (3 pts.) By considering the frequency domain (i.e., DFTs), show that for any two time-domain vectors $\mathbf{x}$ and $\mathbf{y}$ of the same size, $\mathbf{x} \circledast \mathbf{y} = \mathbf{P}\mathbf{x} \circledast \mathbf{P}^{-1}\mathbf{y}$.

(iii) (6 pts.) If

$$\begin{bmatrix} 2 & 2 & 1 & 0 & 1 & -3 \\ -3 & 2 & 2 & 1 & 0 & 1 \\ 1 & -3 & 2 & 2 & 1 & 0 \\ 0 & 1 & -3 & 2 & 2 & 1 \\ 1 & 0 & 1 & -3 & 2 & 2 \\ 2 & 1 & 0 & 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \\ 23 \\ 12 \\ 8 \\ -4 \end{bmatrix},$$

explain how the vector $\mathbf{x}$ can be obtained using DFTs (as opposed to a conventional solution of a 6×6 system of equations). Implement this solution in MATLAB to obtain $\mathbf{x}$.

(iv) (5 pts.) Let $\mathbf{x}$ and $\mathbf{y}$ be vectors of length $N = 14$, given by

$$\begin{aligned} x[n] &= a + b\cos\left(\frac{\pi n}{7} + \phi_1\right) + c\cos\left(\frac{5\pi n}{7} + \phi_3\right) \\ y[n] &= d(-1)^n + e\cos\left(\frac{2\pi n}{7} + \phi_3\right) + f\cos\left(\frac{4\pi n}{7} + \phi_4\right) \end{aligned}$$

where $n = 0, \ldots, 13$. With as little algebra as possible, determine the circular convolution vector $\mathbf{s} = \mathbf{x} \circledast \mathbf{y}$.

Problem 8B

The signal

$$\mathbf{x} = \begin{bmatrix} a & b & c & 0 & 0 & 0 & a & b & c & 0 & 0 & 0 \end{bmatrix}^T$$

has DFT $\mathbf{X}$ given by

$$\mathbf{X} = \begin{bmatrix} D_0 & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & D_7 & D_8 & D_9 & D_{10} & D_{11} \end{bmatrix}^T$$

(i) (4 pts.) For which indices k is D_k necessarily equal to zero?

(ii) (4 pts.) If

$$\mathbf{x}^{(1)} = \begin{bmatrix} a & b & c \end{bmatrix}^T,$$

express the DFT $\mathbf{X}^{(1)}$ in terms of nonzero D_k's.

(iii) (4 pts.) If

$$\mathbf{x}^{(2)} = \begin{bmatrix} a & b & c & a & b & c & a & b & c \end{bmatrix}^T ,$$

express the DFT $\mathbf{X}^{(2)}$ in terms of nonzero D_k's.

(iv) (4 pts.) If

$$\mathbf{x}^{(3)} = \begin{bmatrix} 0 & 0 & 0 & a & b & c \end{bmatrix}^T ,$$

express the DFT $\mathbf{X}^{(3)}$ in terms of nonzero D_k's.

(v) (4 pts.) Let

$$\mathbf{u} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

have DFT $\mathbf{U}$. If $\mathbf{X}^{(4)}$ is the DFT of length $N = 4$ obtained by

- circularly convolving $\mathbf{X}$ and $\mathbf{U}$, followed by
- sampling every third entry of the result (i.e., the circular convolution vector),

express the time-domain vector $\mathbf{x}^{(4)}$ in terms of a, b and c.

Problem 8C

(*Submit both plots.*)

The data set `data08C.txt` contains forty noisy samples of a continuous-time signal which is a sum of two sinusoids at frequencies f_1 and f_2 (Hz), i.e.,

$$s(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2)$$

The sampling rate f_s equals 40 samples/second (i.e., the forty samples correspond to exactly one second in real time) and is greater than the Nyquist rate for this signal.

(i) (4 pts.) If $\mathbf{x}$ is the signal vector in `data08C.txt`, compute and plot the magnitude spectrum of $\mathbf{x}$ using the `BAR` command.

(ii) (6 pts.) Compute the magnitude spectrum obtained after zero-padding $\mathbf{x}$ to length $N = 800$, and plot it using the `PLOT` command.

(iii) (5 pts.) Use the last plot to estimate the values of A_1 and A_2.

(iv) (5 pts.) Use the last plot to estimate the values of f_1 and f_2.

Solved Examples

S **8.1** (*P* **3.20** in textbook). The time-domain signals $\mathbf{x}$ and $\mathbf{y}$ have DFT's

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 1 & -1 \end{bmatrix}^T$$

and

$$\mathbf{Y} = \begin{bmatrix} 3 & 5 & 8 & -4 \end{bmatrix}^T$$

(i) Is either $\mathbf{x}$ or $\mathbf{y}$ real-valued?

(ii) Does either $\mathbf{x}$ or $\mathbf{y}$ have circular conjugate symmetry?

(iii) Without inverting $\mathbf{X}$ or $\mathbf{Y}$, determine the DFT of the signal $\mathbf{s}^{(1)}$ defined by

$$s^{(1)}[n] = x[n]y[n] \ , \qquad n = 0, 1, 2, 3$$

(iv) Without inverting $\mathbf{X}$ or $\mathbf{Y}$, determine the DFT of the signal $\mathbf{s}^{(2)}$ defined by

$$\mathbf{s}^{(2)} = \mathbf{x} \circledast \mathbf{y}$$

S **8.2** (*P* **3.21** in textbook). The time-domain signals

$$\mathbf{x} = \begin{bmatrix} 2 & 0 & 1 & 3 \end{bmatrix}^T$$

and

$$\mathbf{y} = \begin{bmatrix} 1 & -1 & 2 & -4 \end{bmatrix}^T$$

have DFT's $\mathbf{X}$ and $\mathbf{Y}$ given by

$$\mathbf{X} = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 \end{bmatrix}^T$$

and

$$\mathbf{Y} = \begin{bmatrix} Y_0 & Y_1 & Y_2 & Y_3 \end{bmatrix}^T$$

Determine the time-domain signal $\mathbf{s}$ whose DFT is given by

$$\mathbf{S} = \begin{bmatrix} X_0Y_2 & X_1Y_3 & X_2Y_0 & X_3Y_1 \end{bmatrix}^T$$

S **8.3** (*P* **3.26** in textbook). The twelve-point signal

$$\mathbf{x} = \begin{bmatrix} a & b & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

has DFT $\mathbf{X}$ given by

$$\mathbf{X} = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} \end{bmatrix}^T$$

Express the following DFT's in terms of the entries of $\mathbf{X}$:
(i) The DFT $\mathbf{Y}$ of

$$\mathbf{y} = \begin{bmatrix} a & b & c & 0 & 0 & 0 \end{bmatrix}^T$$

(ii) The DFT $\mathbf{S}$ of

$$\mathbf{s} = \begin{bmatrix} a & b & c & a & b & c & a & b & c & a & b & c \end{bmatrix}^T$$

S **8.4** (*P* **3.27** in textbook). In MATLAB notation, consider the 4-point vector

```
s = [a b c d].'
```

and its zero-padded extension

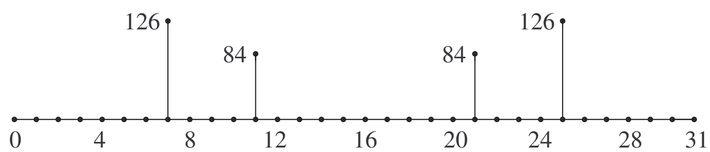
```
x = [s ; zeros(12,1)]
```

Let `X` denote the DFT of `x`. Express the DFT's of the following vectors using the entries of `X`:

```
x1 = s
x2 = [ s ; s ]
x3 = [ s ; s ; s ; s ; s ]  % length=20
x4 = [ s ; z4 ]
x5 = [ z4 ; s ]
x6 = [ s ; z4 ; s ; z4 ]
x7 = [ s ; s ; z4 ; z4 ]
```

where `z4 = zeros(4,1)`.

S **8.5** (*P* **3.29** in textbook). A continuous-time signal consists of two sinusoids at frequencies f_1 and f_2 (Hz). The signal is sampled at a rate of 500 samples/sec (assumed to be greater than the Nyquist rate), and 32 consecutive samples are recorded. The figure shows the graph of magnitude of the 32-point DFT (i.e., without zero-padding) as a function of the frequency index k.



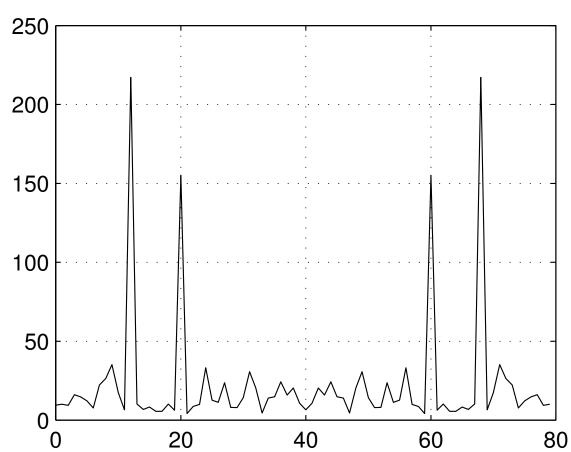
(i) What are the frequencies f_1 and f_2 (in Hz)?

(ii) Is it possible to write an equation for the continuous-time signal based on the information given? If so, write that equation. If not so, explain why.

S **8.6** (*P* **3.30** in textbook). A continuous-time signal is given by

$$x(t) = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2) + z(t)$$

where $z(t)$ is noise. The signal $x(t)$ is sampled every 1.25 ms for a total of 80 samples. The figure shows the graph of the DFT of the 80-point signal as a function of the frequency index $k = 0, \ldots, 79$.



Based on the given graph, what are your estimates of f_1 and f_2?

(You may assume that no aliasing has occurred, i.e., the sampling rate of 800 samples/sec is no less than twice each of f_1 and f_2.)

S **8.7** (*P* **3.31** in textbook). A continuous-time signal is a sum of two sinusoids at frequencies 164 Hz and 182 Hz. 200 samples of the signal are obtained at the rate of 640 samples/sec, and the DFT of the samples is computed.

(i) What frequencies $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ are present in the discrete-time signal obtained by sampling at the above rate?

(ii) Of the 200 Fourier frequencies in the DFT, which two are closest to $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$?

(iii) If the 200 samples are padded with M zeros and the $(M+200)$-point DFT is computed, what would be the least value of M for which both ω_1 and ω_2 are Fourier frequencies?