ENEE 222 0201/2 **HOMEWORK ASSIGNMENT 7** **Due Tue 04/08/14**

Problem 7A

The signal vector $\mathbf{s}$ is given by

$$\mathbf{s} = \begin{bmatrix} a & b & c & d & e & f & g & h \end{bmatrix}^T$$

(i) (8 pts.) Display the following vectors:

- $\mathbf{s}^{(1)} = \mathbf{PRs} + \mathbf{RPs}$
- $\mathbf{s}^{(2)} = \mathbf{P}^3\mathbf{s} + \mathbf{P}^{-3}\mathbf{s}$
- $\mathbf{s}^{(3)} = \mathbf{s} - \mathbf{F}^4\mathbf{s}$
- $\mathbf{s}^{(4)} = \mathbf{F}^2\mathbf{s} + \mathbf{F}^{-2}\mathbf{s}$

(ii) (12 pts.) Express the following vectors in terms of $\mathbf{P}$, $\mathbf{R}$, $\mathbf{F}$ and $\mathbf{s}$:

- $\mathbf{s}^{(5)} = \begin{bmatrix} 0 & b-h & c-g & d-f & 0 & f-d & g-c & h-b \end{bmatrix}^T$
- $\mathbf{s}^{(6)} = \begin{bmatrix} c+g & d+h & e+a & f+b & c+g & d+h & e+a & f+b \end{bmatrix}^T$
- $\mathbf{s}^{(7)} = \begin{bmatrix} 2a & -\sqrt{2}b & 0 & \sqrt{2}d & -2e & \sqrt{2}f & 0 & -\sqrt{2}h \end{bmatrix}^T$

Problem 7B

Let the signal

$$\mathbf{s} = \begin{bmatrix} a & b & c & d & e & f & g & h \end{bmatrix}^T$$

of Homework Problem 7A have DFT

$$\mathbf{S} = \begin{bmatrix} 7 & -2 & 4j & 1+3j & -3 & 1-3j & -4j & -2 \end{bmatrix}^T$$

Without computing any DFTs or inverse DFTs, determine (in numerical form) the DFTs of the signal vectors $\mathbf{s}^{(1)}$ through $\mathbf{s}^{(7)}$ constructed in that problem.

The MATLAB functions `OSHIFT`, `OFLIP` and `FDIAG` found on the course homework web page can be used for circular shifts, circular reversals and products with Fourier sinusoids (by means of $\mathbf{F}$). You may use these functions to verify your results in MATLAB. Just copy these M-files to a local directory and include that directory in the MATLAB path.

Problem 7C

Single-step the MATLAB script below, keeping a figure window open. Answer questions (i)–(iv). Print and submit the two bar plots (as indicated) with your answers.

(Make sure that your MATLAB path includes a directory containing the M-files `oshift.m` and `oflip.m`, found on the course homework web page.)

```
N = 36  ;
n = (0:N-1).' ;

A  = randn(N,1) ;
B  = randn(N,1) ;
bar(A + oflip(A)) ;                     % Label as "Graph 1" and submit
bar(B - oflip(B)) ;                     % Label as "Graph 2" and submit

S = A + oflip(A) + j*(B - oflip(B)) ;
s = ifft(S) ;
norm(imag(s))/norm(real(s))             % see (i) below
s = real(s) ;

S2 = fft(S)    ;                        % see (ii) below

X = real(S)  ;
x = ifft(X)  ;
bar(x)       ;
y = s + oflip(s)    ;
bar(x./y)                               % see (iii) below

x1 = oshift(x,10) ;                     %
x2 = oshift(x,18) ;                     %
x3 = oshift(x,10) + oshift(x,26) ;      %
x4 = x.*cos(n*pi/8) ;                   %
x5 = x.*cos(n*pi/9) ;                   % see (iv) below
```

(i) (3 pts.) *Why* is this value so small?

(ii) (4 pts.) What is the exact algebraic relationship between `S2` and `s`? *Explain.*

(iii) (3 pts.) Why is this graph flat at 0.5?

(iv) (10 pts.) Based on the formulas shown, and without any additional computation or use of MATLAB (other than for verification), determine whether each of the vectors `x1` through `x5` has a real-valued DFT. *Explain your answers.*

Solved Examples

S **7.1** (*P* **3.10** in textbook). Let

$$\mathbf{x} = \begin{bmatrix} 2 & 1 & -1 & -2 & -3 & 3 \end{bmatrix}^T$$

Display (as vectors) and sketch the following signals:

- $\mathbf{x}^{(1)} = \mathbf{P}\mathbf{x}$
- $\mathbf{x}^{(2)} = \mathbf{P}^5\mathbf{x}$

- $\mathbf{x}^{(3)} = \mathbf{P}\mathbf{x} + \mathbf{P}^5\mathbf{x}$
- $\mathbf{x}^{(4)} = \mathbf{x} + \mathbf{R}\mathbf{x}$
- $\mathbf{x}^{(5)} = \mathbf{x} - \mathbf{R}\mathbf{x}$
- $\mathbf{x}^{(6)} = \mathbf{F}^3\mathbf{x}$
- $\mathbf{x}^{(7)} = (\mathbf{I} - \mathbf{F}^3)\mathbf{x}$

S **7.2** (*P* **3.11** in textbook). The time-domain signal

$$\mathbf{x} = \begin{bmatrix} a & b & c & d & e & f \end{bmatrix}^T$$

has DFT

$$\mathbf{X} = \begin{bmatrix} A & B & C & D & E & F \end{bmatrix}^T$$

Using the given parameters, and defining

$$\lambda = \frac{1}{2} \qquad \text{and} \qquad \mu = \frac{\sqrt{3}}{2}$$

for convenience, write out the components of the DFT vector $\mathbf{X}^{(r)}$ for each of the following time-domain signals $\mathbf{x}^{(r)}$.

$$\begin{aligned}
\mathbf{x}^{(1)} &= \begin{bmatrix} a & -b & c & -d & e & -f \end{bmatrix}^T \\
\mathbf{x}^{(2)} &= \begin{bmatrix} a & 0 & c & 0 & e & 0 \end{bmatrix}^T \\
\mathbf{x}^{(3)} &= \begin{bmatrix} a & f & e & d & c & b \end{bmatrix}^T \\
\mathbf{x}^{(4)} &= \begin{bmatrix} d & e & f & a & b & c \end{bmatrix}^T \\
\mathbf{x}^{(5)} &= \begin{bmatrix} b & a & f & e & d & c \end{bmatrix}^T \\
\mathbf{x}^{(6)} &= \begin{bmatrix} f+b & a+c & b+d & c+e & d+f & e+a \end{bmatrix}^T \\
\mathbf{x}^{(7)} &= \begin{bmatrix} 0 & \mu b & \mu c & 0 & -\mu e & -\mu f \end{bmatrix}^T \\
\mathbf{x}^{(8)} &= \begin{bmatrix} A & B & C & D & E & F \end{bmatrix}^T
\end{aligned}$$

S **7.3** (*P* **3.13** in textbook). An eight-point signal $\mathbf{x}$ has DFT

$$\mathbf{X} = \begin{bmatrix} 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \end{bmatrix}^T$$

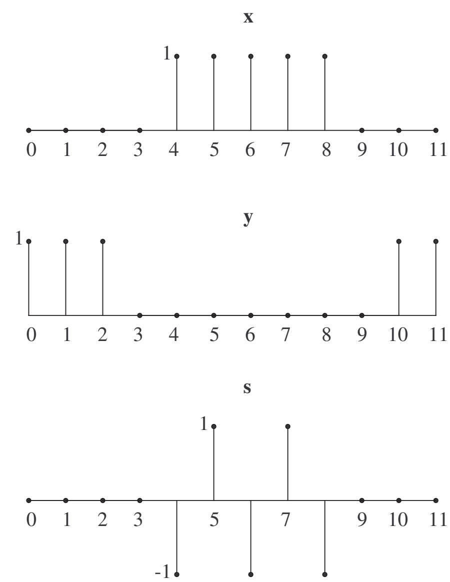
Without inverting $\mathbf{X}$, compute the DFT $\mathbf{Y}$ of $\mathbf{y}$, which is given by the equation

$$y[n] = x[n] \cdot \cos\left(\frac{\pi n}{4}\right), \qquad n = 0, \ldots, 7$$

S **7.4** (*P* **3.15** in textbook). Consider the twelve-point vectors **x**, **y** and **s** shown in the figure. If the DFT **X** is given by

$$\mathbf{X} = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} \end{bmatrix}^T$$

express the DFT's **Y** and **S** in terms of the entries of **X**.

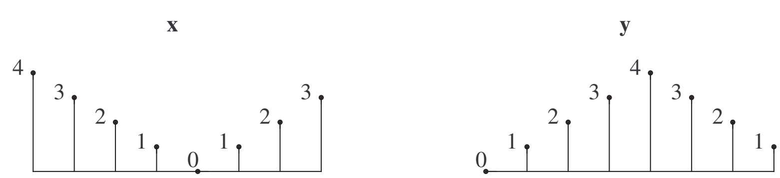


S **7.5** (*P* **3.14** in textbook). Consider the signal **x** shown in the figure (on the left). Its spectrum is given by

$$\mathbf{X} = \begin{bmatrix} A & B & C & D & E & F & G & H \end{bmatrix}^T$$

(i) The DFT vector shown above contains duplicate values. What are those values?

(ii) Express the DFT **Y** of the signal **y** (shown on the right) in terms of the entries of **X**.



S **7.6** (*P* **3.16** in textbook). Run the MATLAB script

```
n = (0:63)';
X =[ones(11,1); zeros(43,1); ones(10,1)];
bar(X), axis tight
max(imag(ifft(X)))      % See (i) below
x = real(ifft(X));
bar(x)
```

```
cs = cos(3*pi*n/4);      % See (ii) below
y = x.*cs;
bar(y);
max(imag(fft(y)))        % See (iii) below
Y = real(fft(y));
bar(Y)                   % See (iv) below
```

(i) Why is this value so small?
(ii) Is this a Fourier sinusoid for this problem?
(iii) Why is this value so small?
(iv) Derive the answer for `Y` analytically, i.e., based on known properties of the DFT.