ENEE 222 0201/2 HOMEWORK ASSIGNMENT 6 Due Tue 04/01/14

Problem 6A

Let

$$\mathbf{V} = \left[\begin{array}{cccccccc} \mathbf{v}^{(0)} & \mathbf{v}^{(1)} & \mathbf{v}^{(2)} & \mathbf{v}^{(3)} & \mathbf{v}^{(4)} & \mathbf{v}^{(5)} & \mathbf{v}^{(6)} & \mathbf{v}^{(7)} \end{array} \right]$$

be the matrix of Fourier sinusoids of length $N = 8$.

(i) (6 pts.) If

$$\mathbf{x} = \left[\begin{array}{cccccccc} 4 & -1 & -2 & -1 & 4 & -1 & -2 & -1 \end{array} \right]^T ,$$

use projections to represent $\mathbf{x}$ in the form $\mathbf{x} = \mathbf{Vc}$. Verify that $\mathbf{x}$ is a linear combination of three columns of $\mathbf{V}$ (only).

(ii) (6 pts.) Repeat for

$$\mathbf{y} = \left[\begin{array}{cccccccc} 2 & 5 & 2 & 1 & 2 & -1 & 2 & 3 \end{array} \right]^T ,$$

expressing it as $\mathbf{y} = \mathbf{Vd}$. Verify that $\mathbf{y}$ is a linear combination of five columns of $\mathbf{V}$.

(iii) (2 pts.) Verify your results in (i) and (ii) using the `FFT` command in MATLAB (which will generate the vectors $8\mathbf{c}$ and $8\mathbf{d}$).

(iv) (2 pts.) Determine the projection $\lambda\mathbf{y}$ of $\mathbf{x}$ onto $\mathbf{y}$.

(v) (4 pts.) If $\mathbf{s} = \mathbf{x} + \mathbf{y}$, use your results from (i) and (ii) above to obtain the least squares approximation $\hat{\mathbf{s}}$ of $\mathbf{s}$ in terms of $\mathbf{v}^{(0)}$, $\mathbf{v}^{(2)}$, $\mathbf{v}^{(4)}$ and $\mathbf{v}^{(6)}$. Display the entries of $\hat{\mathbf{s}}$. Also, compute the squared error norm $\|\mathbf{s} - \hat{\mathbf{s}}\|^2$.

Problem 6B

Solve by hand (no inner products are needed). You may want to verify your answers on MATLAB. All vectors have length $N = 9$.

(i) (4 pts.) The time-domain vector

$$\mathbf{x}^{(1)} = \left[\begin{array}{ccccccccc} 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{array} \right]^T$$

is a Fourier sinusoid. What is its frequency ω? By considering the appropriate column of the Fourier matrix $\mathbf{V}$, determine and display the DFT $\mathbf{X}^{(1)}$.

(ii) (4 pts.) Express the time-domain vector

$$\mathbf{x}^{(2)} = \left[\begin{array}{ccccccccc} 2 & -1 & -1 & 2 & -1 & -1 & 2 & -1 & -1 \end{array} \right]^T$$

as a linear combination of $\mathbf{x}^{(1)}$ (above) and another easily identifiable Fourier sinusoid. Hence determine and display the DFT $\mathbf{X}^{(2)}$.

(iii) (4 pts.) Determine and display the DFT $\mathbf{X}^{(3)}$ of

$$x^{(3)}[n] = \cos(2\pi n/9) + 3\cos(8\pi n/9) , \qquad n = 0, \ldots, 8$$

(iv) (4 pts.) Using the formula $e^{j\theta} - e^{-j\theta} = 2j\sin\theta$, determine and display the DFT $\mathbf{X}^{(4)}$ of

$$x^{(4)}[n] = \sin(4\pi n/9) , \qquad n = 0, \ldots, 8$$

(v) (4 pts.) If the time-domain vector $\mathbf{x}^{(5)}$ has DFT

$$\mathbf{X}^{(5)} = \begin{bmatrix} 9 & 0 & 6j & 3 & -1 & -1 & 3 & -6j & 0 \end{bmatrix}^T$$

write an equation for $x^{(5)}[n]$, where $n = 0, \ldots, 8$. Your equation should be in terms of cosines and sines. (*Also:* $\mathbf{x}^{(5)}$ *should be real-valued.*)

Problem 6C

The real-valued signal vector $\mathbf{s}$ has DFT

$$\mathbf{S} = \begin{bmatrix} 5 & z_1 & z_2 & z_3 & z_4 & -8 & 4+j & -3j & -2-3j & 1 \end{bmatrix}^T$$

(i) (2 pts.) What are the values of z_1, z_2, z_3 and z_4?

(ii) (3 pts.) Without using complex algebra (or MATLAB), determine the value of

$$s[0] + s[2] + s[4] + s[6] + s[8]$$

(iii) (4 pts.) Display the amplitude ($|S[k]|$) and phase ($\angle S[k]$) spectra as vectors.

(iv) (7 pts.) Write an equation for $s[n]$ (where $n = 0 : 9$) in the form of a constant plus five real-valued sinusoids with frequencies between 0 and π (inclusive).

(v) (4 pts.) In MATLAB, generate the vector `s` given by the equation found in part (iv) and verify that `fft(s)` agrees with $\mathbf{S}$.

Solved Examples

S **6.1** (*P* **3.4** in textbook). The columns of the matrix

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

are the complex Fourier sinusoids of length $N = 4$.

Express the vector

$$\mathbf{s} = \begin{bmatrix} 1 & 4 & -2 & 5 \end{bmatrix}^T$$

as a linear combination of the above sinusoids. In other words, find a vector $\mathbf{c} = [c_0\ c_1\ c_2\ c_3]^T$ such that $\mathbf{s} = \mathbf{V}\mathbf{c}$.

S **6.2** (*P* **3.1** in textbook). Let

$$\alpha = \frac{1}{2} \quad \text{and} \quad \beta = \frac{\sqrt{3}}{2}$$

(i) Determine the complex number z such that the vector

$$\mathbf{v} = \begin{bmatrix} 1 & \alpha + j\beta & -\alpha + j\beta & -1 & -\alpha - j\beta & \alpha - j\beta \end{bmatrix}^T$$

equals

$$\begin{bmatrix} 1 & z & z^2 & z^3 & z^4 & z^5 \end{bmatrix}^T$$

(ii) If

$$\mathbf{s} = \begin{bmatrix} 3 & 2 & -1 & 0 & -1 & 2 \end{bmatrix}^T$$

determine the least-squares approximation $\hat{\mathbf{s}}$ of $\mathbf{s}$ in the form of a linear combination of $\mathbf{1}$ (i.e., the all-ones vector), $\mathbf{v}$ and $\mathbf{v}^*$. Clearly show the numerical values of the elements of $\hat{\mathbf{s}}$.

S **6.3** (*P* **3.2** in textbook). Let $\mathbf{v}^{(0)}$, $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(7)}$ denote the complex Fourier sinusoids of length $N = 8$ at frequencies $\omega = 0$, $\omega = \pi/4$ and $\omega = 7\pi/4$, respectively.

Determine the least-squares approximation $\hat{\mathbf{s}}$ of

$$\mathbf{s} = \begin{bmatrix} 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \end{bmatrix}$$

based on $\mathbf{v}^{(0)}$, $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(7)}$. Compute the squared approximation error $\|\mathbf{s} - \hat{\mathbf{s}}\|^2$.

S **6.4** (*P* **3.7** in textbook). A five-point *real-valued* signal $\mathbf{x}$ has DFT given by

$$\mathbf{X} = \begin{bmatrix} 4 & 1+j & 3-j & z_1 & z_2 \end{bmatrix}^T$$

(i) Compute $x[0] + x[1] + x[2] + x[3] + x[4]$ using one entry of $\mathbf{X}$ *only*.

(ii) Determine the values of z_1 and z_2.

(iii) Compute the amplitude and phase spectra of $\mathbf{x}$, displaying each as a vector.

(iv) Express $x[n]$ as a linear combination of three real-valued sinusoids.

S **6.5** (*P* **3.8** in textbook). Consider the real-valued signal $\mathbf{x}$ given by

$$x[n] = 3(-1)^n + 7\cos\left(\frac{\pi n}{4} + 1.2\right) + 2\cos\left(\frac{\pi n}{2} - 0.8\right), \qquad n = 0, \dots, 7$$

(i) Which Fourier frequencies (for an eight-point sample) are present in the signal $\mathbf{x}$?

(ii) Determine the amplitude spectrum of $\mathbf{x}$, displaying your answer in the form

$$\begin{bmatrix} A_0 & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 \end{bmatrix}^T$$

(iii) Determine the phase spectrum of $\mathbf{x}$, displaying your answer in the form

$$\begin{bmatrix} \phi_0 & \phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 & \phi_6 & \phi_7 \end{bmatrix}^T$$