ENEE 222 0201/2 HOMEWORK ASSIGNMENT 5 Due Tue 03/25/15

Problem 5A

Let $\mathbf{v}^{(1)} = [\,1 \quad 2 \quad -2 \quad -1\,]^T$, $\mathbf{v}^{(2)} = [\,3 \quad -1 \quad 3 \quad -5\,]^T$ and $\mathbf{v}^{(3)} = [\,7 \quad -6 \quad -4 \quad 3\,]^T$.

(i) (3 pts.) Show that $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$ are (pairwise) orthogonal, and compute their norms.

In what follows, let $\mathbf{s} = [\,5 \quad 2 \quad -5 \quad 4\,]^T$.

(ii) (3 pts.) Determine the projection $\mathbf{f}^{(i)}$ of $\mathbf{s}$ onto each $\mathbf{v}^{(i)}$ (where $i = 1, 2, 3$).

(iii) (2 pts.) Determine the angle between $\mathbf{s}$ and $\mathbf{v}^{(1)}$.

(iv) (3 pts.) Determine the angle between $\mathbf{s}$ and the plane defined by $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$.

(v) (3 pts.) Determine the projection $\mathbf{g}$ of $\mathbf{s}$ onto the three-dimensional subspace defined by $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$.

(iv) (2 pts.) Verify that error vector $\mathbf{s} - \mathbf{g}$ is orthogonal to each of $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$.

(vii) (4 pts.) Solve the system

$$\begin{bmatrix} 1 & 3 & 7 & 1 \\ 2 & -1 & -6 & 1 \\ -2 & 3 & -4 & 1 \\ -1 & -5 & 3 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 15 \\ 9 \\ -11 \end{bmatrix}$$

without using Gaussian elimination.

Problem 5B

Consider the complex-valued matrix

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}^{(1)} & \mathbf{v}^{(2)} & \mathbf{v}^{(3)} & \mathbf{v}^{(4)} \end{bmatrix} = \begin{bmatrix} 1 & a+jb & 3 & 2+j \\ 2+j & 1 & a+jb & 3 \\ 3 & 2+j & 1 & a+jb \\ a+jb & 3 & 2+j & 1 \end{bmatrix}$$

(i) (6 pts.) Show that there exists only one complex number $a + jb$ such that the columns of $\mathbf{V}$ are pairwise orthogonal. For that choice of $a + jb$, what are the resulting column norms? (*Solve for a and b by setting the inner product of any two columns equal to zero, then verify that the remaining inner products are zero also. Check your answers in MATLAB using* `V'*V` *before proceeding further.*)

From now on, assume that $a + jb$ is as found in part (i) above.

(ii) (6 pts.) Determine $\mathbf{c}$ such that the real-valued vector

$$\mathbf{s} = \begin{bmatrix} 9 & 2 & -9 & -2 \end{bmatrix}^T$$

equals $\mathbf{Vc}$. (*Gaussian elimination is not needed here. Again, verify your answers in MATLAB.*)

(iii) (5 pts.) Determine the projection $\hat{\mathbf{s}}$ of $\mathbf{s}$ onto the subspace generated by the vectors $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$. What is the value of $\|\mathbf{s} - \hat{\mathbf{s}}\|^2$?

(iv) (3 pts.) If

$$\begin{aligned}\mathbf{x} &= \mathbf{v}^{(1)} + 2j\mathbf{v}^{(2)} + \mathbf{v}^{(3)} + 2j\mathbf{v}^{(4)} \\ \mathbf{y} &= \mathbf{v}^{(1)} + 3\mathbf{v}^{(2)} - 3\mathbf{v}^{(3)} - \mathbf{v}^{(4)}\end{aligned}$$

compute $\langle \mathbf{x}, \mathbf{y} \rangle$ without using any of the numerical entries of the vectors $\mathbf{v}^{(i)}$.

Solved Examples

S **5.1** (*P* **2.24** in textbook). Consider the four-dimensional vectors $\mathbf{a} = [\ -1\ \ 7\ \ 2\ \ 4\ ]^T$ and $\mathbf{b} = [\ 3\ \ 0\ \ -1\ \ -5\ ]^T$.

(i) Compute $\|\mathbf{a}\|$, $\|\mathbf{b}\|$ and $\|\mathbf{b} - \mathbf{a}\|$.

(ii) Compute the angle θ between $\mathbf{a}$ and $\mathbf{b}$ (where $0 \le \theta \le \pi$).

(iii) Let $\mathbf{f}$ be the projection of $\mathbf{b}$ on $\mathbf{a}$, and $\mathbf{g}$ be the projection of $\mathbf{a}$ on $\mathbf{b}$. Express $\mathbf{f}$ and $\mathbf{g}$ as $\lambda\mathbf{a}$ and $\mu\mathbf{b}$, respectively (λ and μ are scalars).

(iv) Verify that $\mathbf{b} - \mathbf{f} \perp \mathbf{a}$ and $\mathbf{a} - \mathbf{g} \perp \mathbf{b}$.

S **5.2** (*P* **2.34** in textbook). Consider the matrix

$$\mathbf{V} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

(i) Compute the inner product matrix $\mathbf{V}^T\mathbf{V}$.

(ii) If

$$\mathbf{x} = [\ 0\ \ 1\ \ -2\ \ 3\ ]^T$$

determine a vector $\mathbf{c}$ such that $\mathbf{V}\mathbf{c} = \mathbf{x}$.

S **5.3** Let

$$\mathbf{A} = \begin{bmatrix} -3 & 1 & 3 & 9 \\ 9 & -3 & 1 & 3 \\ 3 & 9 & -3 & 1 \\ 1 & 3 & 9 & -3 \end{bmatrix}$$

(i) Carefully compute $\mathbf{A}^T\mathbf{A}$. What special property do the columns of $\mathbf{A}$ have?

(ii) The vector $\mathbf{b} = [\ 2\ \ -1\ \ 8\ \ -9\ ]^T$ can be written as a linear combination of three columns of $\mathbf{A}$. Determine that linear combination (i.e., its coefficients) *without solving a* 4×4 *system.*

S **5.4** The complex valued-matrix

$$\mathbf{V} = [\ \mathbf{v}^{(1)}\ \ \mathbf{v}^{(2)}\ \ \mathbf{v}^{(3)}\ ] = \begin{bmatrix} 3 & a+jb & 6j \\ 6j & 3 & a+jb \\ a+jb & 6j & 3 \end{bmatrix}$$

(where a and b are real-valued) has pairwise orthogonal columns.

(i) Determine a and b. What are the column norms?

(ii) Express the complex-valued vector

$$\mathbf{s} = \begin{bmatrix} -5-2j & -10+8j & 1-14j \end{bmatrix}^T$$

as a linear combination of $\mathbf{v}^{(1)}$, $\mathbf{v}^{(2)}$ and $\mathbf{v}^{(3)}$.

(iii) If

$$\begin{aligned} \mathbf{x} &= \frac{1}{2}\mathbf{v}^{(1)} - \frac{1}{2}\mathbf{v}^{(2)} + 3\mathbf{v}^{(3)} \\ \mathbf{y} &= 2\mathbf{v}^{(1)} + \frac{3}{2}\mathbf{v}^{(2)} - \mathbf{v}^{(3)} \end{aligned}$$

explain why the projection $\lambda\mathbf{x}$ of $\mathbf{y}$ onto $\mathbf{x}$ is such that λ is real. Determine λ without performing complex operations.