ENEE 222 0201/2 HOMEWORK ASSIGNMENT 3 Due Tue 02/25/14

Problem 3A

(i) (4 pts.) Consider the frequency $f_0 = 420$ Hz and the sampling rate $f_s = 600$ samples/sec. List all the aliases of f_0 with respect to f_s in the frequency range 0.0 to 3.0 kHz.

(ii) (2 pts.) If we sample, at rate $f_s = 600$ samples/sec, a continuous-time real-valued sinusoid whose frequency is one of the values found in part (i) above, what is the resulting frequency ω of the sample sequence? Give your answer in the range $[0, \pi]$ (radians/sample).

(iii) (5 pts.) The discrete-time sinusoid $x[n] = 2.9\cos(0.4\pi n - 0.7)$ is obtained by sampling a continuous-time sinusoid $x(t)$ at a rate of 600 samples per second. If it is known that the frequency of $x(t)$ is in the range 600 to 900 Hz, write an equation for $x(t)$.

(iv) (4 pts.) Let $x[n]$ be as in (iii), with the sampling rate unchanged. If, instead, it is known that the frequency of $x(t)$ is in the range 1,500 to 1,800 Hz, write a new equation for $x(t)$.

(v) (5 pts.) Consider the three continuous-time signals

$$x_1(t) = \cos(36\pi t)\ , \qquad x_2(t) = \cos(56\pi t) \qquad \text{and} \qquad x_3(t) = \cos(116\pi t)$$

The three signals are sampled at the same rate $f_a = 1/T_a$ to produce the sequences $x_i[n] = x_i(nT_a)$ (where $i = 1, 2, 3$). Determine the only value of f_a greater than 7 samples/second such that the three sequences are identical, i.e.,

$$x_1[n] \ = \ x_2[n] \ = \ x_3[n] \qquad \text{(all } n\text{)}$$

Problem 3B

(i) (10 pts.) It is known that

$$\mathbf{A}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{u}, \qquad \mathbf{A}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \mathbf{v} \qquad \text{and} \qquad \mathbf{A}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \mathbf{w}$$

Express

$$\mathbf{A}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. Can you determine the dimensions (size) of $\mathbf{A}$ based on this information?

(ii) (5 pts.) If

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and $\mathbf{Bx} = [\,-7 \ \ 3 \ \ 0 \ \ 2 \ \ -5 \ \ 1\,]^T$, determine the vector $\mathbf{x}$.

(iii) (5 pts.) Study the function `FFTSHIFT` in MATLAB. Ignore the references to spectra and frequencies, and focus on how `FFTSHIFT` transforms a vector of odd or even length. Based on your observations, find the matrix $\mathbf{C}$ which is equivalent to `FFTSHIFT`, i.e., `C*x` equals `fftshift(x)`. Do so in the cases where `length(x)` equals 7 and 8.

Solved Examples

S **3.1** (*P* **1.21** in textbook). For what frequencies f in the range 0 to 3.0 KHz does the sinusoid

$$x(t) = \cos(2\pi f t)$$

yield the signal

$$x[n] = \cos(0.4\pi n)$$

when sampled at a rate of $f_s = 1/T_s = 800$ samples/sec?

S **3.2**. The continuous-time sinusoid

$$x(t) = \cos(2\pi f t + 1.8)$$

is such that $700 < f \leq 800$ (Hz). It is sampled every $T_s = 10.0$ ms to produce the discrete-time sinusoid $x[n] = x(nT_s)$.

(i) If $x[n] = \cos(0.7\pi n + 1.8)$, what is the value of f?

(ii) If, on the other hand, $x[n] = \cos(0.4\pi n - 1.8)$, what is the value of f?

S **3.3** (*P* **1.22** in textbook). The continuous-time sinusoid

$$x(t) = \cos(300\pi t)$$

is sampled every $T_s = 2.0$ ms, so that

$$x[n] = x(0.002n)$$

(i) For what other values of f in the range 0 Hz to 2.0 KHz does the sinusoid

$$v(t) = \cos(2\pi f t)$$

produce the same samples as $x(t)$ (i.e., $v[\cdot] = x[\cdot]$) when sampled at the same rate?

(ii) If we increase the sampling period T_s (or equivalently, drop the sampling rate), what is the least value of T_s greater than 2 ms for which $x(t)$ yields the same sequence of samples (as for $T_s = 2$ ms)?

S **3.4** (*P* **2.3** in textbook). If

$$\mathbf{B}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{B}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix},$$

determine

$$\mathbf{B}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

What are the dimensions of the matrix $\mathbf{B}$?

S **3.5**. (*P* **2.4** in textbook). Let $\mathbf{G}$ be a $m \times 2$ matrix such that

$$\mathbf{G}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \mathbf{u} \qquad \text{and} \qquad \mathbf{G}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \mathbf{v}$$

Express

$$\mathbf{G}\begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

in terms of $\mathbf{u}$ and $\mathbf{v}$.

S **3.6**. (*P* **2.1** in textbook). In MATLAB, enter the matrix

```
A = [1 2 3 4; 5 6 7 8; 9 10 11 12]
```

(i) Find two-element arrays `I` and `J` such that `A(I,J)` is a 2×2 matrix consisting of the corner elements of `A`.

(ii) Suppose that `B=A` initially. Find two-element arrays `K` and `L` such that

```
B(:,K) = B(:,L)
```

swaps the first and fourth columns of `B`.

(iii) Explain the result of

```
C = A(:)
```

(iv) Study the function `RESHAPE`. Use it together with the transpose operator `.'` in a single (one-line) command to generate the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{bmatrix}$$

from `A`.

S **3.7** (*P* **2.5** in textbook). The transformation $A : \mathbf{R}^3 \mapsto \mathbf{R}^3$ is such that

$$\mathbf{A}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}, \qquad \mathbf{A}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}, \qquad \mathbf{A}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

Write out the matrix $\mathbf{A}$ and compute $\mathbf{Ax}$, where

$$\mathbf{x} = \begin{bmatrix} 2 & 5 & -1 \end{bmatrix}^T$$

(ii) The transformation $B : \mathbf{R}^3 \mapsto \mathbf{R}^2$ is such that

$$\mathbf{B}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}, \qquad \mathbf{B}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \qquad \mathbf{B}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Determine the matrix $\mathbf{B}$.