ENEE 222 0201/2 HOMEWORK ASSIGNMENT 2 Due Tue 02/18/14

Problem 2A

Consider the discrete-time sinusoids

$$x[n] = \cos\left(\frac{8\pi n}{15} + \frac{\pi}{6}\right) \qquad \text{and} \qquad y[n] = \cos\left(\frac{\sqrt{5}\pi n}{2} + \frac{\pi}{6}\right)$$

(i) (3 pts.) Which of the two sinusoids is periodic, and what is its fundamental period?

(ii) (4 pts.) Use MATLAB to generate separate plots of $x[n]$ and $y[n]$ for $n = 0, \ldots, 44$.

(iii) (4 pts.) Let N be the value of the period found in part (i). For what values of ω in $[0, \pi]$ is

$$\cos(\omega n)$$

periodic with fundamental period N?

(iv) (3 pts.) An equivalent form for $x[\,\cdot\,]$ is

$$x[n] = \cos(\omega n + \phi)$$

where ω is between π and 2π. What are the values of ω and ϕ?

(v) (2 pts.) The sequence $v[\,\cdot\,]$ is formed by taking every third sample in $x[\,\cdot\,]$, i.e.,

$$v[n] = x[3n]$$

Write an equation for $v[n]$. What is the period of $v[\,\cdot\,]$?

(vi) (4 pts.) Using phasors, express

$$x[n] + 4x[n-1] + x[n-2]$$

as a single real-valued sinusoid.

Problem 2B

Two periods of the sinusoid $x(t) = A\cos(\Omega t + \phi)$ are plotted below. The value of $x(0)$ equals $-4\cos(2\pi/5)$.

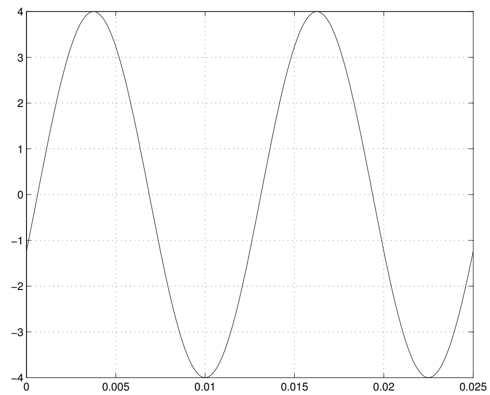
(i) (6 pts.) Determine A, Ω and ϕ. Express ϕ as an exact rational multiple of π in the range $[0, 2\pi)$.

In what follows: A and ϕ are as found in part (i) and $x[n] = x(nT_s)$, where T_s is a suitably chosen sampling period.

(ii) (3 pts.) Determine all values of T_s such that $x[n] = -x[n-1]$ for all n.

(iii) (3 pts.) Determine all values of T_s such that $x[n] = A\cos((3\pi/4)n + \phi)$.

(iv) (4 pts.) Determine all values of T_s such that $x[n] = x[n+8]$ for all n.



(v) (4 pts.) Determine all values of T_s such that $x[n] = A\cos((2\pi/5)n - \phi)$.

Solved Examples

S **2.1**. Find all frequencies ω in $[0, \pi]$ for which the discrete-time sinusoid

$$x[n] \;=\; \cos \omega n$$

is periodic with *fundamental* period $N = 16$.

S **2.2** (*P* **1.15** in textbook). **(i)** For exactly *one* value of ω in $[0, \pi]$, the discrete-time sinusoid

$$v[n] = A\cos(\omega n + \phi)$$

is periodic with period equal to $N = 4$ time units. What is that value of ω?

(ii) For that value of ω, suppose the first period of $v[n]$ is given by

$$v[0] = 1, \qquad v[1] = 1, \qquad v[2] = -1 \qquad \text{and} \qquad v[3] = -1$$

What are the values of $A > 0$ and ϕ?

S **2.3** (*P* **1.17** in textbook). **(i)** Use MATLAB to plot four periods of the discrete-time sinusoid

$$x_1[n] = \cos\left(\frac{7\pi n}{9} + \frac{\pi}{6}\right)$$

(ii) Show that the product

$$x_2[n] = x_1[n] \cdot \cos(\pi n)$$

is also a (real-valued) discrete-time sinusoid. Express it in the form $A\cos(\omega n + \phi)$, where $A > 0$, $\omega \in [0, \pi]$ and $\phi \in (0, 2\pi)$.

S **2.4** (*P* **1.16** in textbook; more difficult). **(i)** Use the trigonometric identity

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

to show that

$$\cos(\omega(n+1)+\phi)+\cos(\omega(n-1)+\phi)=2\cos(\omega n+\phi)\cos\omega$$

(ii) Suppose

$$x[1]=1.7740, \quad x[2]=3.1251 \quad \text{and} \quad x[3]=0.4908$$

are three consecutive values of the discrete-time sinusoid $x[n]=A\cos(\omega n+\phi)$, where $A>0$, $\omega\in[0,\pi]$ and $\phi\in[0,2\pi]$. Use the equation derived in **(i)** to evaluate ω. Then use the ratio $x[2]/x[1]$ together with the given identity for $\cos(\alpha+\beta)$ to evaluate $\tan\phi$ and hence ϕ. Finally, determine A.

S **2.5** (*P* **1.19** in textbook). The continuous-time sinusoid

$$x(t)=\cos(150\pi t+\phi)$$

is sampled every $T_s=3.0$ ms starting at $t=0$. The resulting discrete-time sinusoid is

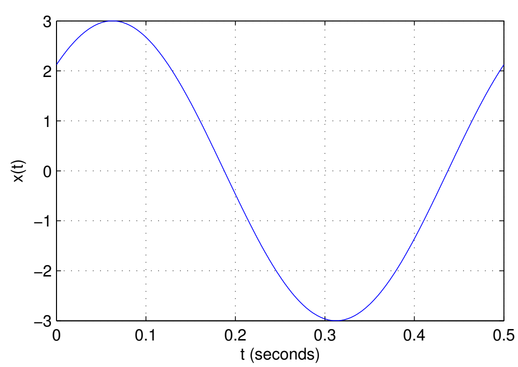
$$x[n]=x(nT_s)$$

(i) Express $x[n]$ in the form $x[n]=\cos(\omega n+\phi)$ i.e., determine the value of ω.

(ii) Is the discrete-time sinusoid $x[n]$ periodic? If so, what is its period?

(iii) Suppose that the sampling rate $f_s=1/T_s$ is variable. For what values of f_s is $x[n]$ constant for all n? For what values of f_s does $x[n]$ alternate in value between $-\cos\phi$ and $\cos\phi$?

S **2.6**. The first period of the sinusoid $x(t)=A\cos(\Omega t+\phi)$ is plotted below, where $x(0)=3\sqrt{2}/2$.



(i) Determine the values of A, Ω and ϕ.

(ii) The sinusoid is sampled every $T_s=0.05$ seconds starting at $t=0$ to produce

$$x[n]=x(nT_s)$$

Write an equation for $x[n]$. Is $x[n]$ periodic, and if so, what is its period?

(iii) What is the relationship between the vector $(x[0],\dots,x[4])$ and the vector $(x[5],\dots,x[9])$?

S **2.7** (this problem has a somewhat different flavor). The continuous time sinusoid $x(t)=\cos(1200\pi t)$ is sampled every T_s seconds starting at time $t=0$. The value of T_s is chosen so that every zero of $x(\cdot)$ is also a (zero-valued) sample in $x[\cdot]$.

(i) What are the possible values of T_s?

(ii) What is the maximum value of T_s (less than one-quarter period of $x(\cdot)$) such that the difference between two consecutive samples does not exceed 0.01 (in absolute value)?