ENEE 222 0201/2 HOMEWORK ASSIGNMENT 1 Due Tue 02/11/14

Problem 1A
Consider the complex numbers

$$z_1 = 4 - 5j \qquad \text{and} \qquad z_2 = -2 + 7j$$

(i) (2 pts.) Plot both numbers on the complex plane.

(ii) (2 pts.) Evaluate $|z_i|$ and $\angle z_i$ for both values of i ($i = 1, 2$).

(iii) (6 pts.) Express each of $z_1 + 3z_2$, $z_1^* + 2z_2$ and z_1^2 in both Cartesian and polar form.

(iv) (3 pts.) If $v = z_1^3 \cdot z_2^{-1}$, determine $|v|$ and $\angle v$. Also, obtain v in Cartesian form.

(v) (4 pts.) If $w = z_2^{3000}$, determine $\angle w$ in the range $[0, 2\pi)$. Your answer should be correct to four decimal places.

(vi) (3 pts.) Determine the only real values b and c so that $z_1^2 + bz_1 + c = 0$.

Problem 1B
Throughout this problem, let

$$u = \sqrt{3} + j \qquad \text{and} \qquad v = -j$$

(i) (2 pts.) Express each number in the form $re^{j\theta}$. Plot both numbers on the complex plane.

(ii) (4 pts.) Let a be real. Determine, in terms of a, the real and imaginary parts of the complex number

$$w = \frac{a}{u} + v$$

For what value of a is w purely real?

(iii) (5 pts.) Determine the constant c such that u is a root of

$$z^9 - c = 0$$

Plot *all* roots of this equation on the complex plane.

(iv) (3 pts.) Sketch the circle described by the equation

$$|z - u| = 2$$

Determine the points of intersection (if any) of this circle and the two axes, real and imaginary.

(v) (3 pts.) Sketch the line described by the equation

$$|z - v| = |z + v|$$

(vi) (3 pts.) At which points does the line

$$|z - v| = |z + 2jv|$$

intersect the real and imaginary axes?

Problem 1C
Consider the sinusoid $x(t) = A\cos(\Omega t + \phi)$, where $A > 0$ and $\phi \in (-\pi, \pi]$. Time t is in seconds. It is known that

- $x(0) = -1.7$;
- $x(0.014) = -1.7$;
- $x(t) \neq -1.7$ for $0 < t < 0.014$;
- the smallest positive value of t for which $x(t) = 0$ is $t = 0.016$.

(i) (4 pts.) Plot the three points $(0, -1.7)$, $(0.014, -1.7)$ and $(0.016, 0)$ on the $(t, x(t))$ plane. Based on the information given and your knowledge of sinusoids, sketch the approximate form of $x(t)$ over the time interval $0 < t < 0.016$, noting any symmetries. What can you say about the value of $x(-0.002)$?

(ii) (4 pts.) Determine the period and angular frequency Ω of $x(t)$.

(iii) (4 pts.) Determine the initial phase ϕ of $x(t)$ as a fraction of π.

(iv) (4 pts.) Determine the amplitude A.

(v) (4 pts.) During a single cycle, how long does it take for the signal value to rise from $-A/4$ to $3A/4$?

Solved Examples

S **1.1** (*P* **1.9** in textbook). Simplify the complex fraction

$$\frac{(1 + j\sqrt{3})(1 - j)}{(\sqrt{3} + j)(1 + j)}$$

using (i) Cartesian forms; and (ii) polar forms, throughout your calculation.

S **1.2** Consider the complex number

$$z = \frac{1 + 2j}{3 - jb}$$

Determine the value(s) of b for which **(i)** z is purely real; **(ii)** z is purely imaginary; **(iii)** z has modulus equal to $\sqrt{2}/2$.

S **1.3** If $z = (3/5) + j(9/10)$, express z^{17} in polar form (r, θ), where $\theta \in [0, 2\pi)$.

S **1.4** On the complex plane, sketch the curves given by the following equations:

(i) $|z - 1 + j| = 1$

(ii) $|z - 2| = |z - 3j|$

Using geometry, determine the minimum value of $|z|$ in each case (i.e., as z traces out each curve).

S **1.5** (*P* **1.10** in textbook). Let N be an arbitrary positive integer. Evaluate the product

$$\prod_{k=1}^{N-1} \left(\cos\left(\frac{k\pi}{N}\right) + j \sin\left(\frac{k\pi}{N}\right) \right) ,$$

expressing your answer in Cartesian form. (Use the identity $1+2+\cdots+n = n(n+1)/2$ to simplify the answer first.)

S **1.6**. Show that the expression

$$f(\theta) = e^{-j2\theta} - e^{-j\theta} + 1 - e^{j\theta} + e^{j2\theta} \ ,$$

where θ ranges over $[0, 2\pi)$, is real-valued, and obtain an alternative expression for it in terms of sines and/or cosines.

S **1.7**. Clearly,

$$e^{j3\theta} = \left(e^{j\theta}\right)^3$$

By expanding $(\cos\theta + j\sin\theta)^3$ and separating real and imaginary parts, obtain two identities: one for $\cos 3\theta$ in terms of powers of $\cos\theta$ only, and another for $\sin 3\theta$ in terms of powers of $\sin\theta$ only.

S **1.8**. (more difficult). Consider two complex numbers w and z, where $w \neq 0$ is fixed and z is variable such that $|z| = 1$. Show that as z traces out the unit circle, the ratio

$$\frac{|z - w^*|}{|z - (1/w)|}$$

is constant in value. (*Hint*: If $|z| = 1$, then $1/z = z^*$.)

S **1.9** (*P* **1.11** in textbook). Consider the continuous-time sinusoid

$$x(t) = 5\cos(500\pi t + 0.25)$$

where t is in seconds.

(i) What is the first value of t greater than 0 such that $x(t) = 0$?

(ii) Consider the following MATLAB script which generates a discrete approximation to $x(t)$:

```
t = 0 : 0.0001 : 0.01 ;
x = 5*cos(500*pi*t + 0.25) ;
```

For which values of `n`, if any, is `x(n)` zero?

S **1.10** (*P* **1.12** in textbook). The value of the continuous-time sinusoid $x(t) = A\cos(\Omega t + \phi)$ (where $A > 0$ and $0 \leq \phi < 2\pi$) is between -2.0 and $+2.0$ for 70% of its period.

(i) What is the value of A?

(ii) If it takes 300 ms for the value of $x(t)$ to rise from -2.0 to $+2.0$, what is the value of Ω?

(iii) If $t = 40$ ms is the first positive time for which $x(t) = -2.0$ and $x'(t)$ (the first derivative) is negative, what is the value of ϕ?

S **1.11** (*P* **1.13** in textbook). The input voltage $v(t)$ to a light-emitting diode circuit is given by $A\cos(\Omega t + \phi)$, where $A > 0$, $\Omega > 0$ and ϕ are unknown parameters. The circuit is designed in such a way that the diode turns on at the moment the input voltage exceeds $A/2$, and turns off when the voltage falls below $A/2$.

(i) What percentage of the time is the diode on?

(ii) Suppose the voltage $v(t)$ is applied to the diode at time $t = 0$. The diode turns on instantly, turns off at $t = 1.5$ ms, then turns on again at $t = 9.5$ ms. Based on this information, determine Ω and ϕ.